Handout for
Addressing Policy Questions With Formal Demography
Formal Demography Workshop
UC Berkeley, June 2022
and at the same time a User Guide for my packages
`AnnualWorldPopulation`, `WorldPopQuery`, `ProjMatRawData`, and `StablePop`

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June 10, 2022

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0 Preliminaries

Please note that the code does not imply any warranties. I need to thoroughly check everything still before it can be actual production code. If you want to use it, please check for yourself before you include any results in your publications or elsewhere.

1 Introduction: Goal of This Document

2 Required Software

All code presented here is written in R (2022). We will use data from the Human Mortality Database ("HMD", 2022) and the Human Fertility Database ("HFD", 2022). This will be done implicity via the packages `HMDHFDplus` by Tim Riffe (2015) as the actual code to access both databases is implemented in my code. Nevertheless, you need to register first with both databases at `www.mortality.org` and `www.humanfertility.org`. And to make things smoother, I would recommend to have the package already installed. In case you do not have it yet, here is the code (obviously, you can use any "repo" you like):

```
install.packages("HMDHFDplus", repos="https://cran.wu.ac.at/")

## Installing package into '/home/roland/R/x86_64-pc-linux-gnu-library/4.2'
## (as 'lib' is unspecified)
```

Loading the package is then done implicitly in my package. But please remember your respective usernames and passwords. They consist of the email address you used for registering and a numeric code used as a password.

I saved them in a file called `accessHMDHFD.txt` in another directory (as a data frame with four columns and one row of data).

```
access.data <- read.table("../../accessHMDHFD.txt", header=TRUE, sep=",")
HMD.user <- trimws(access.data$HMDuser[1],"b")
HMD.pw <- trimws(access.data$HMDpassword[1],"b")
HFD.user <- trimws(access.data$HFDuser[1],"b")
HFD.pw <- trimws(access.data$HFDpassword[1],"b")
```

You will need those later on. Please remember then that you have to use your own credentials instead of `HMD.user` or `HMD.pw`, for instance `frank.drebin@example.com` and `123456`. Please do not forget that the password — despite being a number — should passed as a character vector (i.e. a string).

My own software is (still) hosted on GitLab.

To load the packages easily from there, you need to have the package `remotes` installed. In case you do not, please:

```
install.packages("remotes", repos="https://cran.wu.ac.at/")

## Installing package into '/home/roland/R/x86_64-pc-linux-gnu-library/4.2'
## (as 'lib' is unspecified)
```

The packages you will need from there are:

```
library(remotes)
```

```
install_gitlab("rolandrau/AnnualWorldPopulation")
install_gitlab("rolandrau/WorldPopQuery")
install_gitlab("rolandrau/ProjMatRawData")
install_gitlab("rolandrau/StablePop")
```

This might take a few seconds.

- Package `AnnualWorldPopulation` is a dependency which is still required. But I am currently thinking of trying to rewrite my code so it is not required anymore. Nevertheless, you might be interested in the package because it allows you to extract population data for single ages and single calendar years from the United Nations' World Population Prospects (2019 Revision, estimates & medium variant). Here is an example:

  ```
  library(AnnualWorldPopulation)
  data(pop.array)
  str(pop.array)

  ##  num [1:101, 1:151, 1:3, 1:255] 295 274 256 240 227 ...
  ##  - attr(*, "dimnames")=List of 4
  ##   ..$ Age   : chr [1:101] "0" "1" "2" "3" ...
  ##   ..$ Period: chr [1:151] "1950" "1951" "1952" "1953" ...
  ##   ..$ Sex   : chr [1:3] "Both" "Female" "Male"
  ##   ..$ Code  : chr [1:255] "4" "8" "12" "24" ...
  ```

  The fourth dimension (`Code`) will be explained in the next section. The code for the US is 840. Hence it is easy to extract the female population aged 80–85 in the year 2000. Either you do it directly:

  ```
  example1 <- pop.array[paste(80:85), paste(2000), "Female", "840"]
  ```

  Or you use the extraction function I wrote:

  ```
  example2 <- population(CountryCode=840, Sex="Female", Ages=80:85, Years=2000)
  ```

  Both should yield the same data (in thousands):

  ```
  example1
  ```

```
##      80      81      82      83      84      85
## 732.298 680.086 626.170 571.398 516.506 461.329
```

```
example2
```

```
##      80      81      82      83      84      85
## 732.298 680.086 626.170 571.398 516.506 461.329
```

- Most people do not know the required country codes which are used by the United Nations. That is why I wrote package `WorldPopQuery`. It allows you to find the code for a country (or region, ...) and vice versa. Some examples might clarify what it can do (please note that upper-/lower-case does not matter and partial matching as well without any wildcards such as $*$, ...).

```
library(WorldPopQuery)
Search.CountryCode("United")
```

```
##                          country countrycode
## 44   United Republic of Tanzania         834
## 106         United Arab Emirates         784
## 165 United States Virgin Islands         850
## 231               United Kingdom         826
## 255     United States of America         840
```

```
Search.CountryCode("NITeD")
```

```
##                          country countrycode
## 44   United Republic of Tanzania         834
## 106         United Arab Emirates         784
## 165 United States Virgin Islands         850
## 231               United Kingdom         826
## 255     United States of America         840
```

And the reverse operation:

```
Search.Country("900")
```

```
##   country countrycode
## 1   WORLD         900
```

```
Search.Country("276")
```

```
##     country countrycode
## 249 Germany         276
```

- Finally, package `ProjMatRawData` contains already the data you need to construct population projection matrices ("Leslie matrices").

```
library(ProjMatRawData)
```

The data are three-dimension arrays (Period × Age × CountryCode) for $L(x)$, $f(x)$, the sex-ratio at birth and the actual population. The names of the objects are `Lx.array`, `fx.array`, `srb.array`, and `pop.array5`, respectively.

Once again, you can access those data directly:

```
str(pop.array5)

## Error in str(pop.array5):  object 'pop.array5' not found

pop.array5["2000-2005", paste(c(20, 25)), "840"]

## Error in eval(expr, envir, enclos):  object 'pop.array5' not found
```

to obtain the female (!) population in the US ("840") in 2000–2005 at ages 20–24 and 25–29 in thousands. Its actual purpose is to support the package `StablePop` (that is why it contains only the female population).

3 The Main Package `StablePop`

The main package we are interested in is the package `StablePop`. It allows the estimation of standard parameters from stable population theory.

```
install_gitlab("rolandrau/StablePop")
```

```
library(StablePop)
```

It contains primarily two functions:

- `stablepopHMDHFD`
- `stablepopUN`

Both functions perform the same analyses (by and large), require different input, though. The function `stablepopHMDHFD` estimates the stable population theory parameters using data from the Human Mortality Database and the Human Fertility Database whereas `stablepopUN` uses data from the World Population Prospects (2019 Revision) of the United Nations. A further difference is that the HMDHFD approach estimates its values for single years and single ages whereas the UN uses 5 calendar years × 5 age years.

3.1 Input

3.1.1 stablepopHMDHFD()

The function `stablepopHMDHFD()` requires the following input paramters:

```
args(stablepopHMDHFD)

## function (country, year, srb = 1.06, HMD.user, HMD.pw, HFD.user,
##     HFD.pw, brute.force.years = 1000)
## NULL
```

country: A character string for the country you want to analyze. If you are unsure about the country codes you need, please check:

```
HMDHFDplus::getHMDcountries()

##  [1] "AUS"     "AUT"     "BEL"     "BGR"     "BLR"     "CAN"     "CHL"
##  [8] "HRV"     "HKG"     "CHE"     "CZE"     "DEUTNP"  "DEUTE"   "DEUTW"
## [15] "DNK"     "ESP"     "EST"     "FIN"     "FRATNP"  "FRACNP"  "GRC"
## [22] "HUN"     "IRL"     "ISL"     "ISR"     "ITA"     "JPN"     "KOR"
## [29] "LTU"     "LUX"     "LVA"     "NLD"     "NOR"     "NZL_NP"  "NZL_MA"
## [36] "NZL_NM"  "POL"     "PRT"     "RUS"     "SVK"     "SVN"     "SWE"
## [43] "TWN"     "UKR"     "GBR_NP"  "GBRTENW" "GBRCENW" "GBR_SCO" "GBR_NIR"
## [50] "USA"

HMDHFDplus::getHFDcountries()

##  [1] "AUT"     "BLR"     "BEL"     "BGR"     "CAN"     "CHL"     "HRV"
##  [8] "CZE"     "DNK"     "EST"     "FIN"     "FRATNP"  "DEUTNP"  "DEUTW"
## [15] "DEUTE"   "HUN"     "ISL"     "ITA"     "JPN"     "LTU"     "NLD"
## [22] "NOR"     "POL"     "PRT"     "KOR"     "RUS"     "SVK"     "SVN"
## [29] "ESP"     "SWE"     "CHE"     "TWN"     "UKR"     "GBR_NP"  "GBRTENW"
## [36] "GBR_SCO" "GBR_NIR" "USA"
```

`year`: A numeric scalar. Calendar year.

`srb`: A numeric scalar. Sex ratio at birth. Default value: 1.06

`HMD.user`: Type character: Username for the Human Mortality Database (Registration Email Address)

`HMD.pw`: Type character: Password for the Human Mortality Database

`HFD.user`: Type character: Username for the Human Fertility Database (Registration Email Address)

`HFD.pw`: Type character: Password for the Human Fertility Database

`brute.force.years`: A numeric scalar. The number of years one wants to project the population into the future by brute force for the population momentum.

3.1.2 `stablepopHMDHFD()`

The function `stablepopUN()` requires the following input paramters:

```
args(stablepopUN)

## function (country = 276, period = "2015-2020")
## NULL
```

`country`: A numeric scalar. Find the respective code with `WorldPopQuery::SearchCountry("Norway")`, for instance

`period`: Type character. Valid time periods are "1950–1955", ... "2095–2100".

3.2 Calling the Functions

Both functions yield lists of almost identical components as output. I will refer to the small difference. Let's estimate our parameters for the United States in 2000 (HMDHFD) and for Brazil 2010–2015 (UN).

Please understand that I do not share my usernames and passwords.

```
access.data <- read.table("../../accessHMDHFD.txt", header=TRUE, sep=",")
HMD.user <- trimws(access.data$HMDuser[1],"b")
HMD.pw <- trimws(access.data$HMDpassword[1],"b")
HFD.user <- trimws(access.data$HFDuser[1],"b")
HFD.pw <- trimws(access.data$HFDpassword[1],"b")
```

```
USA.HMDHFD <- stablepopHMDHFD(country="USA", year=2010,
  HMD.user=trimws(access.data$HMDuser[1],"b"),
  HMD.pw=trimws(access.data$HMDpassword[1],"b"),
  HFD.user=trimws(access.data$HFDuser[1],"b"),
  HFD.pw=trimws(access.data$HFDpassword[1],"b"))
```

```
Search.CountryCode("Brazil")

##     country countrycode
## 178  Brazil          76

BRA.UN <- stablepopUN(country=76, period="2005-2010")
```

The most important publication for the code is, without any doubt: Caswell (2001).

3.3 Output

The precise description of the output can be obtained by

```
str(USA.HMDHFD)
str(BRA.UN)
```

Here is a (hopefully) more useful explanation.

`projmat` The actual projection matrix. Either a 111×111 matrix (HMDHFD) or a 21×21 matrix.

`projmatMomentum` The projection matrix after fertility has been switched to replacement level (NRR=1, classic momentum case). Either a 111×111 matrix (HMDHFD) or a 21×21 matrix.

`growth.rate` The long-term/intrinsic growth rate of the population. Estimated via the dominant eigenvalue.

```
USA.HMDHFD$growth.rate

## [1] 0.9971286

BRA.UN$growth.rate

## [1] 0.984972
```

`age.structure.now` The current age structure.

`age.structure.stable` The age structure of the stable population corresponding to the given fertility and survival values.

`age.structure.momentum` The age structure in the momentum case, i.e. when NRR is set to 1.

`NRR` Net reproduction rate.

```
USA.HMDHFD$NRR

## [1] 0.9216791

BRA.UN$NRR

## [1] 0.9230773
```

which is pretty close to the "official" NRR for Brazil of 0.917 (UN WPP 2019).

`gen.T` Generation length

```r
USA.HMDHFD$gen.T
```

```
## [1] 28.36253
```

```r
BRA.UN$gen.T
```

```
## [1] 5.286074
```

```r
BRA.UN$gen.T * 5
```

```
## [1] 26.43037
```

`m.bar` Mean age of the age-specific fertility rates

```r
USA.HMDHFD$m.bar
```

```
## [1] 28.33211
```

```r
BRA.UN$m.bar
```

```
## [1] 5.278803
```

```r
BRA.UN$m.bar * 5
```

```
## [1] 26.39401
```

`mu.1` Mean age at childbearing in the stationary population

```r
USA.HMDHFD$mu.1
```

```
## [1] 28.309
```

```r
BRA.UN$mu.1
```

```
## [1] 5.270559
```

```r
BRA.UN$mu.1 * 5
```

```
## [1] 26.3528
```

`A.bar` Mean age at childbearing in the stable population

```
USA.HMDHFD$A.bar

## [1] 28.41611

BRA.UN$A.bar

## [1] 5.301633

BRA.UN$A.bar * 5

## [1] 26.50817
```

`ex` Remaining life expectancy at age *x*.

`e0` Life Expectancy at birth.

```
USA.HMDHFD$e0

##
## 81.47506

BRA.UN$e0

##
## 15.61438

BRA.UN$e0 * 5

##
## 78.07192
```

`growth.rate.sensitivity` perturbation analysis of the long term growth rate: sensitivity matrix

`growth.rate.elasticity` perturbation analysis of the long term growth rate: elasticity matrix

`reproduction.value` R.A. Fisher's reproductive value, scaled in a way $ReproductiveValue[1]=1

`Momentum.PrestonGuillot` Population momentum according to the analytic equation of Preston and Guillot (1997)

```r
USA.HMDHFD$Momentum.PrestonGuillot

## [1] 1.06808

BRA.UN$Momentum.PrestonGuillot

## [1] 1.343618
```

`Momentum.bruteforce` Population momentum estimated via "brute force"

```r
USA.HMDHFD$Momentum.bruteforce

## [1] 1.067946

BRA.UN$Momentum.bruteforce

## [1] 1.343659
```

`Momentum.bruteforce` Population momentum estimated via "brute force" using my own recursive implementation.

```r
USA.HMDHFD$MomentumRecursive

## [1] 1.068069

BRA.UN$MomentumRecursive

## [1] 1.343616
```

`RecursiveCalls` provides simply the number of recursive calls to find the stopping condition (not really needed. But potentially useful for debugging.)

4 Platform and Software

The software I am using should be platform independent. In case you want to compare your setup with mine, here is the "Session Info". The document itself has been prepared using package `knitr` (as you will see later for yourself), version 1.39.

```
sessionInfo()

## R version 4.2.0 (2022-04-22)
## Platform: x86_64-pc-linux-gnu (64-bit)
## Running under: Ubuntu 22.04 LTS
##
## Matrix products: default
## BLAS:   /usr/local/lib/R/lib/libRblas.so
## LAPACK: /usr/local/lib/R/lib/libRlapack.so
##
## locale:
##  [1] LC_CTYPE=en_US.UTF-8       LC_NUMERIC=C
##  [3] LC_TIME=de_DE.UTF-8        LC_COLLATE=en_US.UTF-8
##  [5] LC_MONETARY=de_DE.UTF-8    LC_MESSAGES=en_US.UTF-8
##  [7] LC_PAPER=de_DE.UTF-8       LC_NAME=C
##  [9] LC_ADDRESS=C               LC_TELEPHONE=C
## [11] LC_MEASUREMENT=de_DE.UTF-8 LC_IDENTIFICATION=C
##
## attached base packages:
## [1] stats     graphics  grDevices utils     datasets  methods   base
##
## other attached packages:
## [1] StablePop_1.0.3           HMDHFDplus_1.9.17
## [3] ProjMatRawData_1.0.3      WorldPopQuery_1.0
## [5] AnnualWorldPopulation_1.0 remotes_2.4.2
## [7] knitr_1.39
##
## loaded via a namespace (and not attached):
##  [1] codetools_0.2-18 XML_3.99-0.9     digest_0.6.29    R6_2.5.1
##  [5] lifecycle_1.0.1  magrittr_2.0.3   evaluate_0.15    highr_0.9
##  [9] httr_1.4.3       cli_3.3.0        rlang_1.0.2      stringi_1.7.6
## [13] curl_4.3.2       xml2_1.3.3       tools_4.2.0      stringr_1.4.0
## [17] tinytex_0.39     xfun_0.31        compiler_4.2.0   rvest_1.0.2
```

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