Interpreting Changes in Life Expectancy During Temporary Mortality Shocks

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Aim

Change in period life expectancy at birth (ΔPLEB) one of several potential measures of mortality shocks (Goldstein & Lee 2020)

Favored by many demographers: (a) “pure” measure of mortality change (independent of age structure), (b) more appealing unit (years per person) than other such measures, (c) comparability

PLEB attractive b/c intuitive interpretation, but that interpretation <- thought experiment with no mortality change

=> Can we derive intuition for ΔPLEB compatible with changing mortality conditions?

The period life table as a synthetic cohort

- In real (closed) cohort:

$$e_0^o = \frac{\int_0^\infty {}^cD(a).ada}{\int_0^\infty {}^cD(a)da}$$

=> average age at death of cohort members

- In period life table, synthetic cohort "mimics" real cohort:

$$e_0^o = \frac{\int_0^\infty d(a).ada}{\int_0^\infty d(a)da} = \frac{\int_0^\infty l(a).\mu(a).ada}{\int_0^\infty l(a).\mu(a)da}$$

=> expected age at death if subjected to $\mu(a)$ throughout lifetime

Difference in PLEB

- Pollard (1988: 266):

$$(e_0^o)^2 - (e_0^o)^1 = \int_0^\infty \left(\mu^1(a) - \mu^2(a)\right).\ {}_a p_0^2 . (e_a^o)^1 da$$

- Rewrite as:

$$(e_0^o)^2 - (e_0^o)^1 = -\frac{\int_0^\infty \left(\mu^2(a) - \mu^1(a)\right). l^2(a). (e_a^o)^1 da}{\int_0^\infty \mu^2(a). l^2(a) da}$$

In numerator, additional decrements by age in mortality regime #2 x life expectancy at that age in regime #1; in denominator all decrements in regime #2

Illustration: A classic

- Age-invariant mortality change: $\left(\mu^2(a) - \mu^1(a)\right) = \delta.\mu^1(a)$
- Symmetry in Pollard's equation =>

$$(e_0^o)^1 - (e_0^o)^2 = -\frac{\int_0^\infty \left(\mu^1(a) - \mu^2(a)\right).l^1(a).(e_a^o)^2 da}{\int_0^\infty \mu^1(a).l^1(a)da}$$

- Rearrange as:

$$(e_0^o)^2 - (e_0^o)^1 = -\delta.\frac{\int_0^\infty \mu^1(a).l^1(a).(e_a^o)^2 da}{\int_0^\infty l^1(a)da}.\frac{\int_0^\infty l^1(a)da}{\int_0^\infty \mu^1(a).l^1(a)da}$$

$$= -\delta.\frac{\int_0^\infty \mu^1(a).l^1(a).(e_a^o)^2 da}{\int_0^\infty l^1(a)da}.(e_0^o)^1$$

A classic (cont.)

- Middle ratio related to Keyfitz' entropy (Goldman & Lord 1986):

$$H^1 = \frac{\int_0^\infty d^1(a).(e_a^o)^1 da}{\int_0^\infty l^1(a) da}$$

- First term in Taylor expansion:

$$(e_0^o)^2 - (e_0^o)^1 \simeq -\delta. H^1.(e_0^o)^1 = -\delta.(e_0^o)^\dagger$$

$(e_0^o)^\dagger$ known as "e-dagger" (Vaupel & Canudas-Romo 2003)

Interpretation of ΔPLEB in synthetic-cohort framework

$$(e_0^o)^2 - (e_0^o)^1 = -\frac{\int_0^\infty \left(\mu^2(a) - \mu^1(a)\right).\, l^2(a).\, (e_a^o)^1 da}{\int_0^\infty \mu^2(a).\, l^2(a) da}$$

- Interpretable as additional years lived by average individual between two time-invariant mortality regimes
- Works for hypothetical “permanent” changes (e.g., classic ex.)
- Problematic for temporary mortality changes (“shocks”)

The period life table as a stationary population

- Compare PLEB:

$$e_0^o = \frac{\int_0^\infty l(a).\mu(a).ada}{\int_0^\infty l(a).\mu(a)da}$$

- & actual mean age at death (MAD) in the population:

$$\overline{a_D} = \frac{\int_0^\infty N(a).\mu(a).ada}{\int_0^\infty N(a).\mu(a)da}$$

=> PLEB an "internally" age-standardized (stationary-equivalent of) MAD

Issues with externally age-standardized MAD

- Externally age-standardized MAD unchanged by proportional change in $\mu(a)$
- Intuition (mortality ↘ should => ↗ MAD) not entirely wrong b/c mortality ↘ should => older age distribution (some exceptions)
- External age-standardization also removes changes in age-structure induced by phenomenon of interest
- Internal age-standardization allows age distribution to change as a result of, and only of, mortality change

ΔPLEB as an internally standardized measure

$$(e_0^o)^2 - (e_0^o)^1 = -\frac{\int_0^\infty (\mu^2(a) - \mu^1(a)).\, l^2(a).\, (e_a^o)^1 da}{\int_0^\infty \mu^2(a).\, l^2(a) da}$$

stationary equivalent of:

$$-\frac{\int_0^\infty (\mu^2(a) - \mu^1(a)).\, N^2(a).\, (e_a^o)^1 da}{\int_0^\infty \mu^2(a).\, N^2(a) da}$$

- In mortality shocks, $(\mu^2(a) - \mu^1(a)).\, N^2(a)$ = $D^E(a)$ “excess deaths” at age *a*, define Mean Unfulfilled Lifespan (*MUL*) as:

$$MUL = \frac{\int_0^\infty D^E(a).\, (e_a^o)^1 da}{D^2}$$

Related measures of premature mortality: YLL

- Different approaches to Years of Life Lost (YLL) to cause of death *c*. Universal:

$$YLL^c = \int_0^\infty D^c(a).(e_a^o)^U da$$

Advantage: additive across populations => global estimates

- Less realistic though, for practical examples, may prefer population-specific:

$$YLL^c = \int_0^\infty D^c(a).(e_a^o)^P da$$

MUL & YLL

- In population-specific approach, define YLL to excess mortality following mortality shock as:

$$YLL^E = \int_0^\infty D^E(a).(e_a^o)^1 da$$

- Related to MUL:

$$MUL = \frac{YLL^E}{D^2}$$

MUL & AYLL

- YLL depends on population size, needs averaging to compare across populations or time periods
- Various denominators used to define Average YLL (AYLL) : per YLL to all causes, per death from all causes, per death from that cause. Using the latter, define Population-Specific AYLL for excess mortality as:

$$PAYLL^E = \frac{\int_0^\infty D^E(a).(e_a^o)^1 da}{\int_0^\infty D^E(a) da} = \int_0^\infty \frac{D^E(a)}{\int_0^\infty D^E(a) da}.(e_a^o)^1 da$$

- => $MUL = \frac{D^E}{D^2}.PAYLL^E$

MUL & P-Score

- P-Score measures relative incidence of excess v. counterfactual (expected) mortality:

$$P = \frac{\int_0^\infty (\mu^2(a) - \mu^1(a)).N^2(a)da}{\int_0^\infty \mu^1(a).N^2(a)da}$$

- P-score related to ratio of excess to all (actual) deaths:

$$\frac{D^E}{D^2} = \frac{P}{1+P}$$

- => $MUL = \frac{P}{1+P}.PAYLL^E$

Summary

- Thinking of ΔPLEB in "forward looking" framework of expectancy (synthetic cohort) problematic for mortality shocks
- "Backward looking" framework maybe more useful: -ΔPLEB stationary equivalent of period population measure, the Mean Unfulfilled Lifespan

1) Interpretable as the mean (per actual death) reduction in longevity (due to mortality shock) in recent death cohort

2) Value depends on $PAYLL^E$ (average YLL per excess deaths) & on P-score (ratio of excess to expected deaths)

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