

Formal demography as tool to innovate measures & concepts of lifespan and aging

June 9th, 2022

Annette Baudisch

Compare Life Span & Aging across Populations

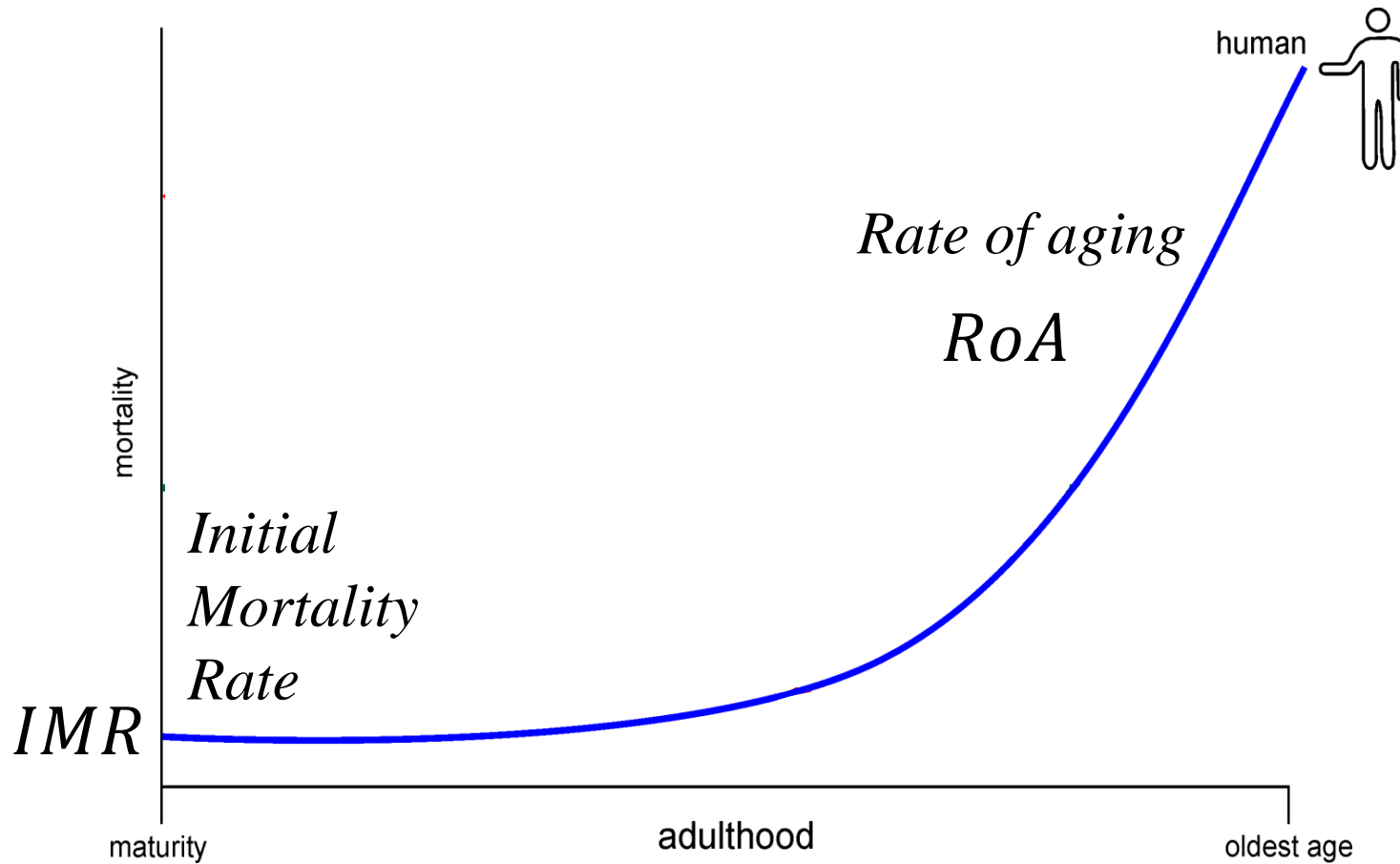


Do the
measures I use
capture all the
signals that are
relevant?
?



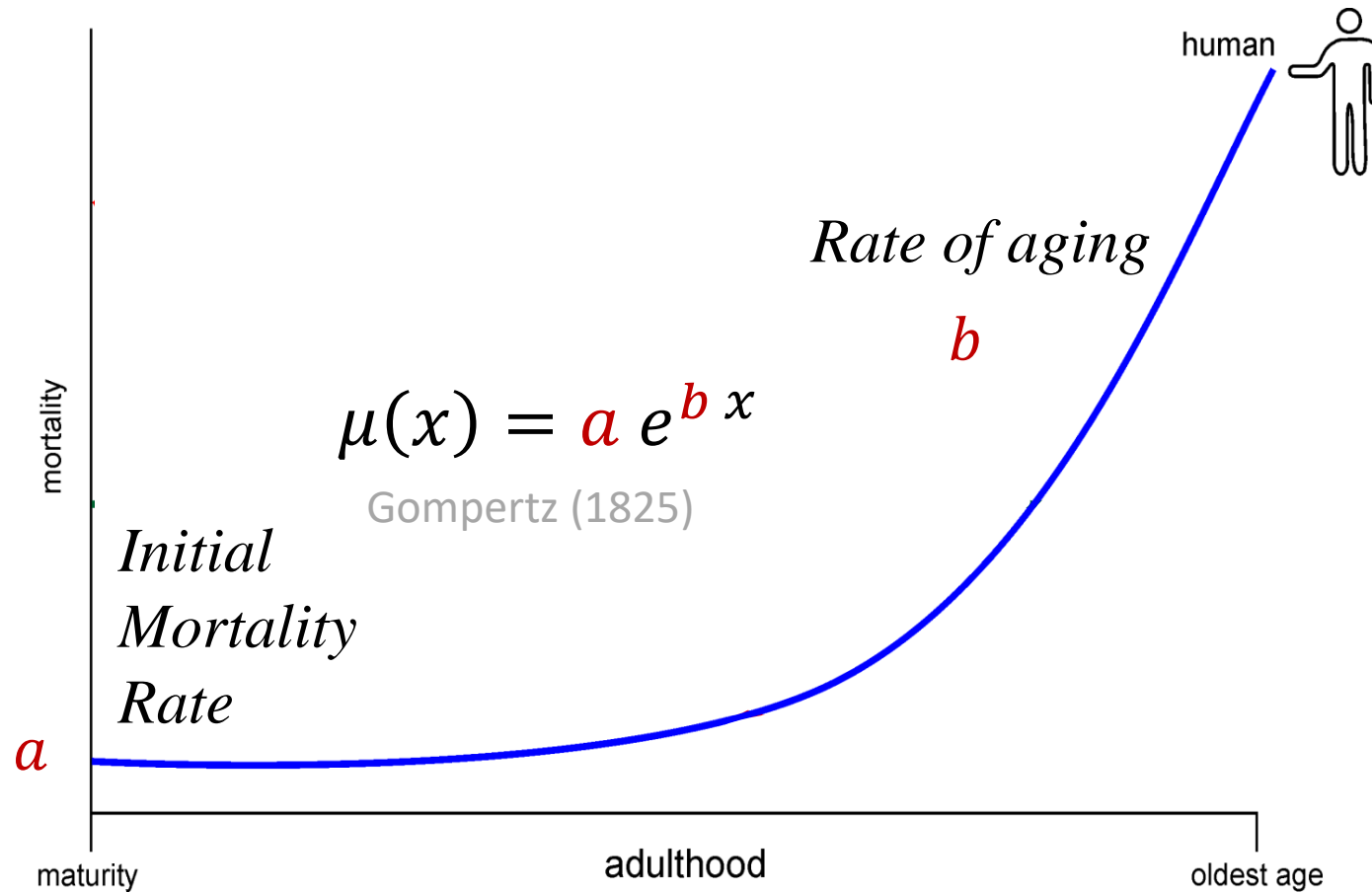
Formal
demography
as strong &
useful tool

Traditional Measures



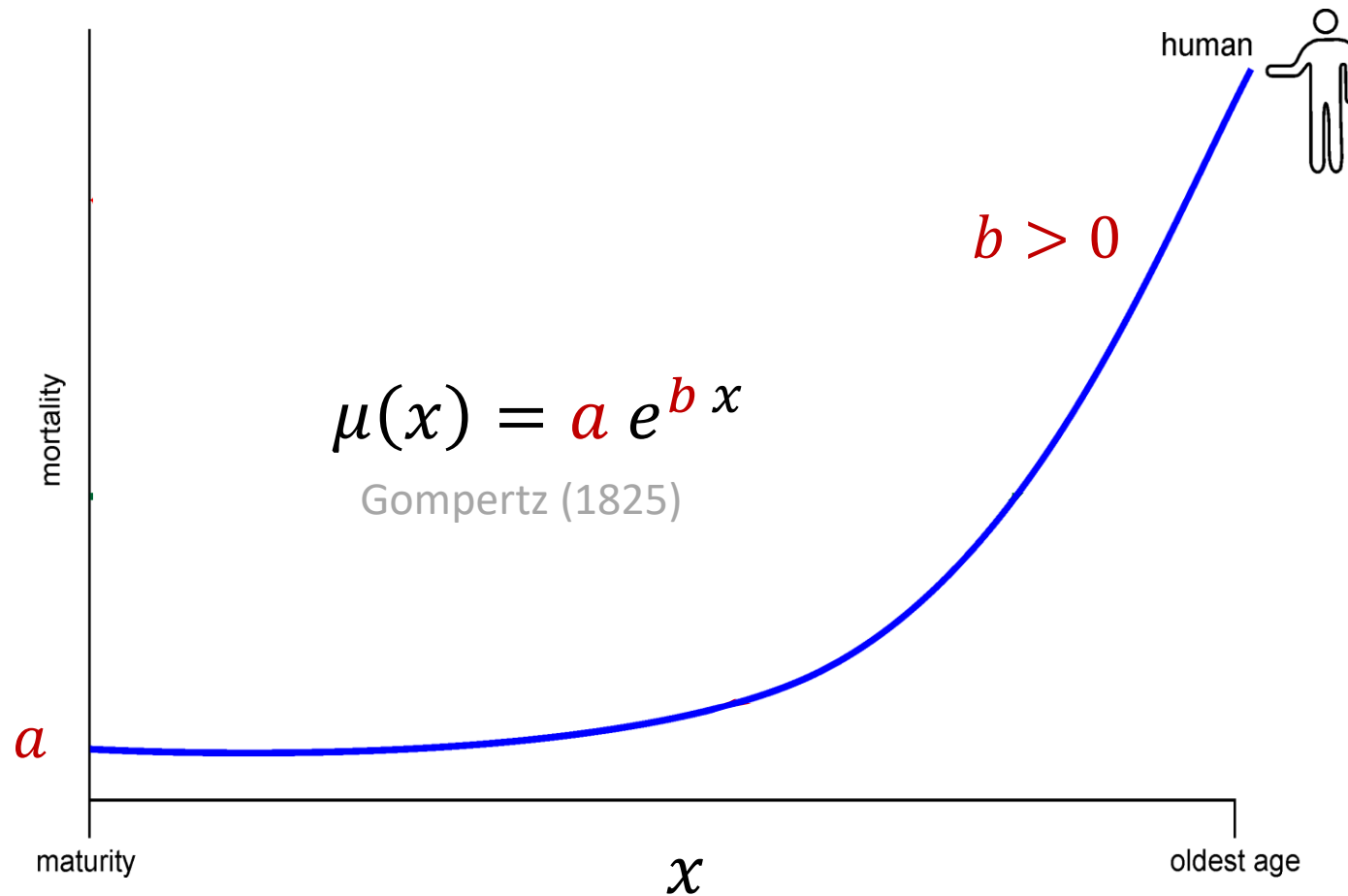
Measures to describe & classify aging patterns

Traditional Measures



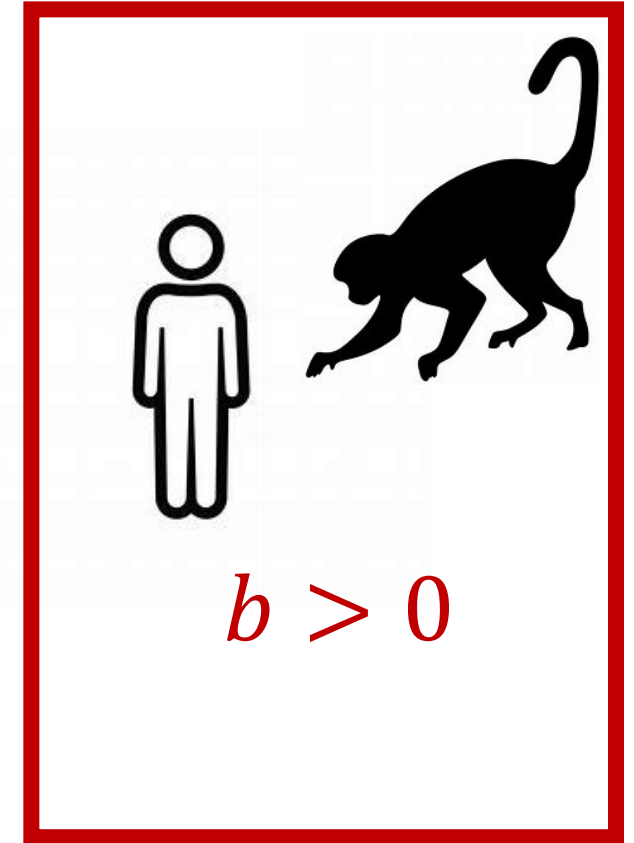
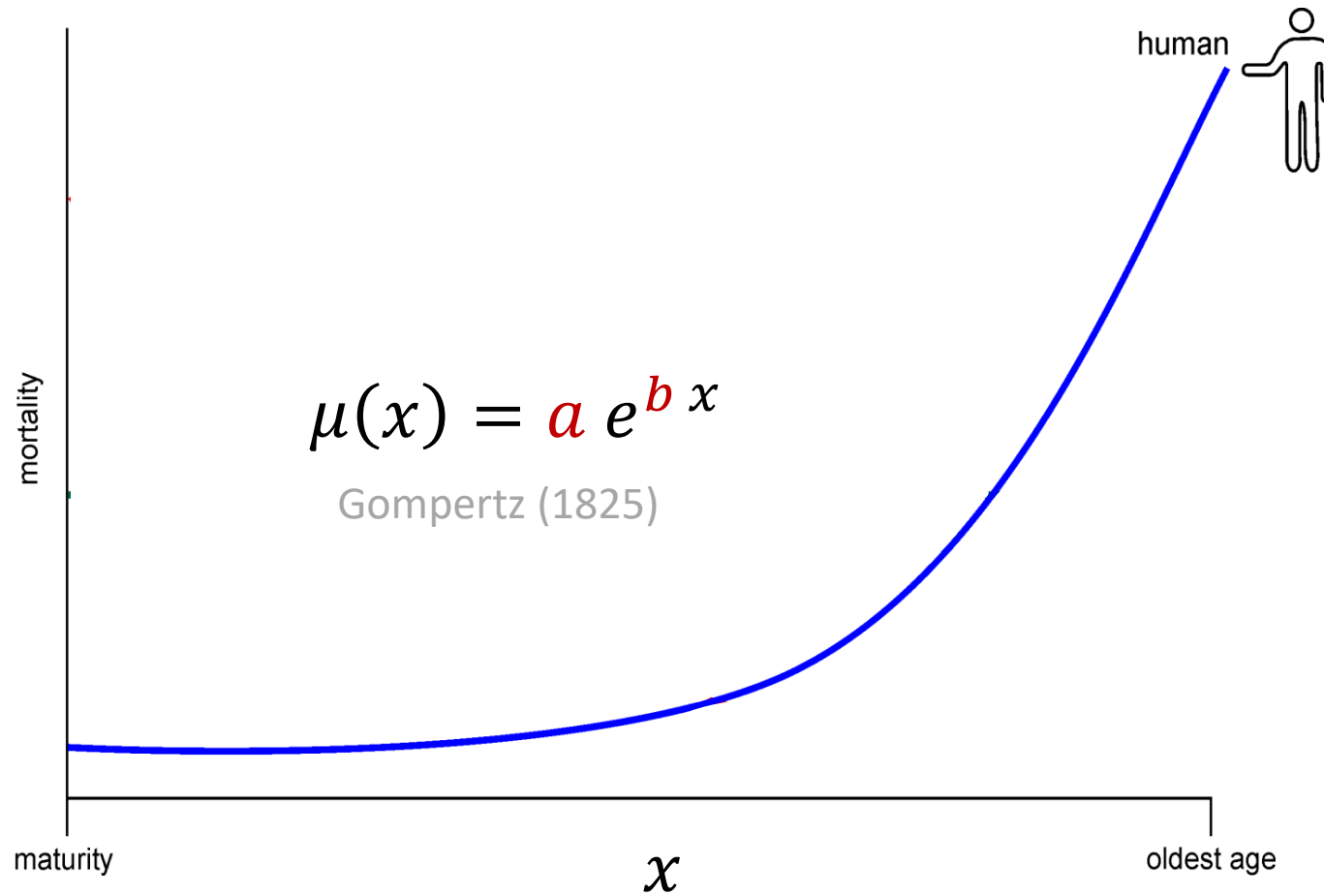
Measures to describe & classify aging patterns

Traditional Measures

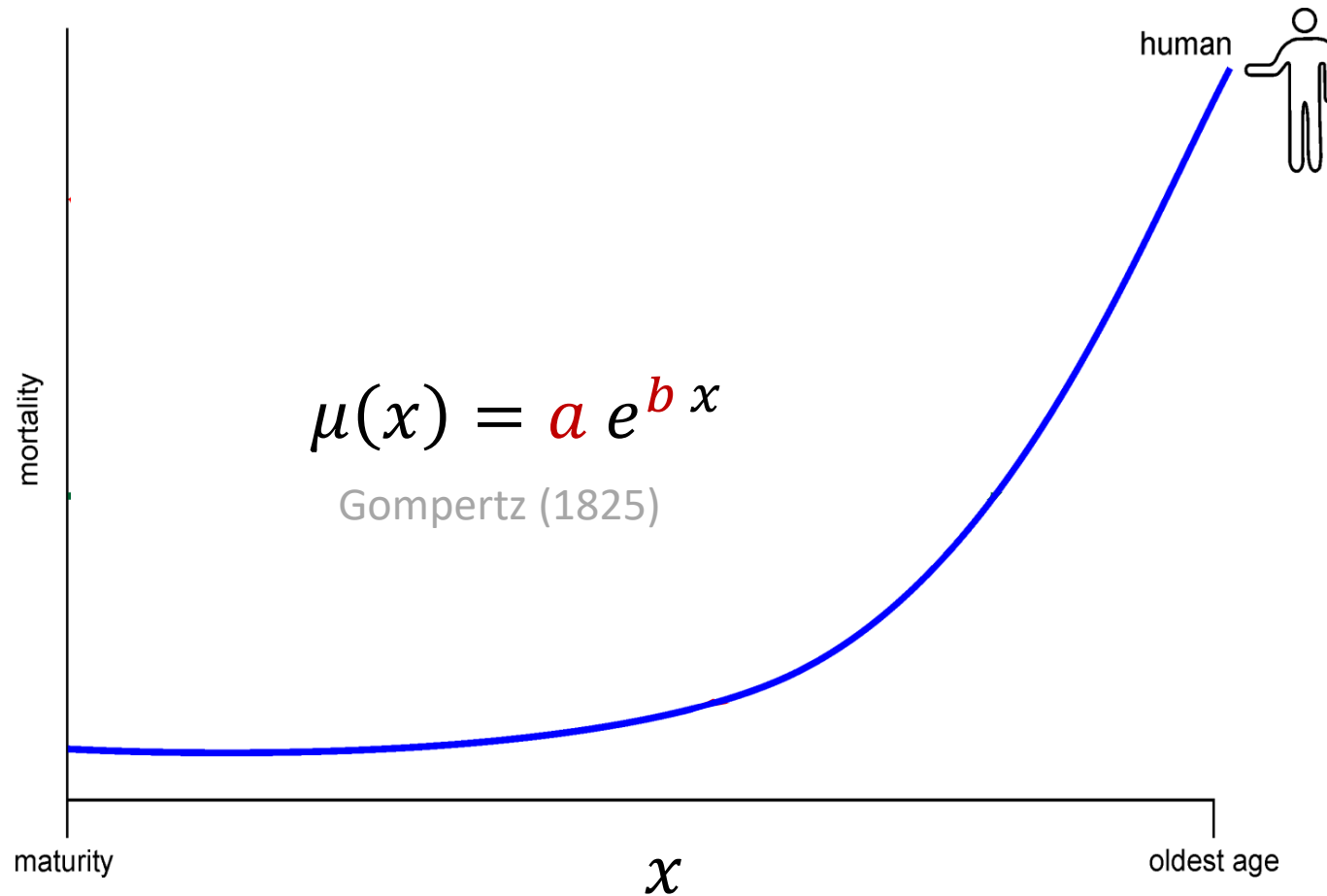


Mathematical
function to
describe
mortality

Traditional Measures



Traditional Measures



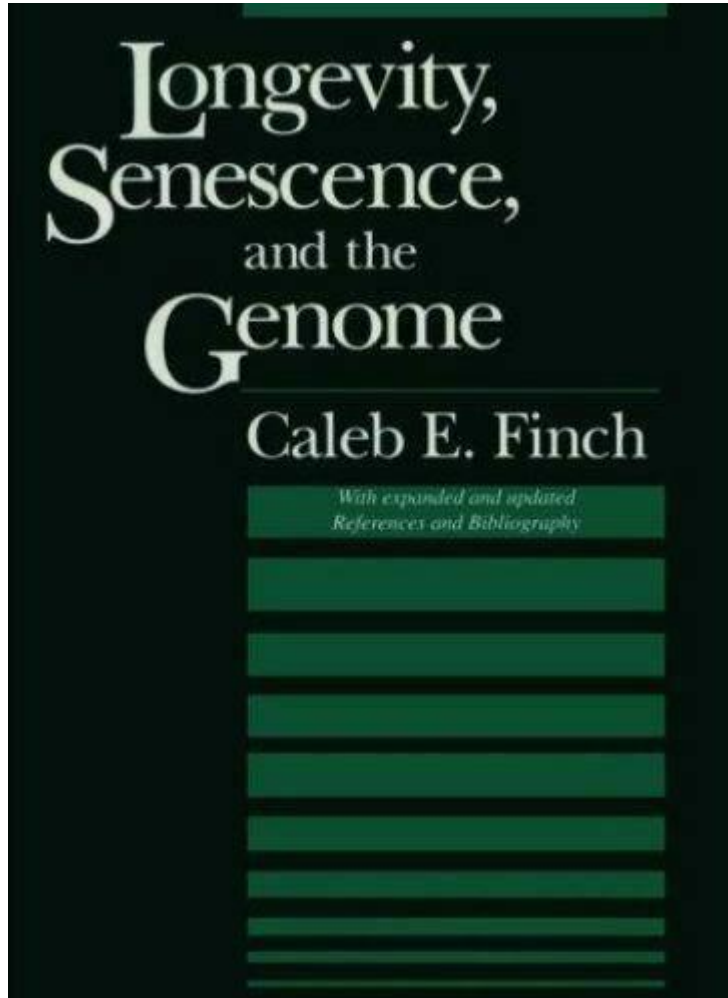
Finch et al. 1990.
Science
**(humans, mammals,
birds)**

Bronikowski et al. 2002.
PNAS
(humans, baboons)

Bronikowski et al. 2011.
Science
(humans, primates)

Empirical diversity of aging patterns

Finch 1990

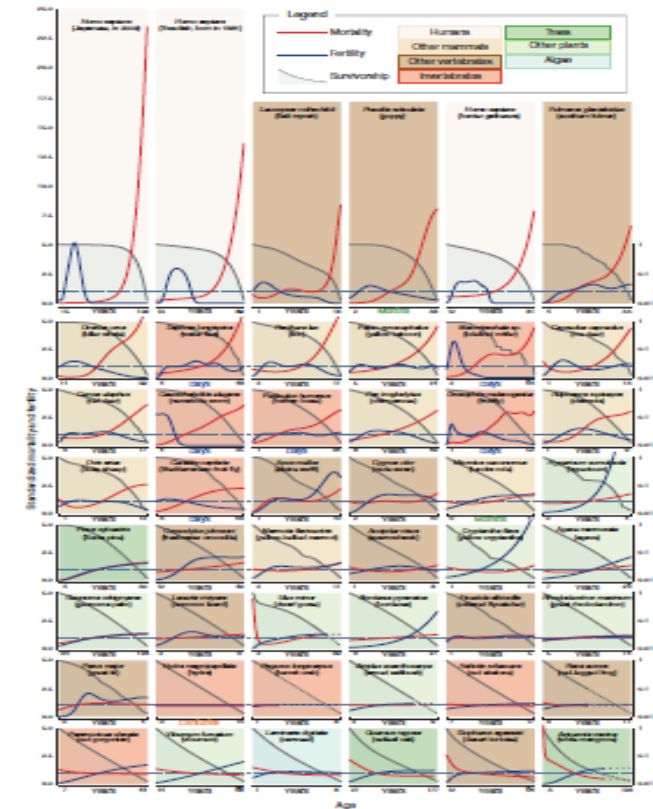


Jones et al 2014 Nature

doi:10.1038/nature12789

Diversity of ageing across the tree of life

Owen R. Jones^{1,2*}, Alexander Scheuerlein^{3*}, Roberto Salguero-Gómez^{3,4}, Carlo Giovanni Camarda⁵, Ralf Schaible³, Brenda B. Casper⁶, Johan P. Dahlgren^{1,2}, Johan Ehrlén⁷, María B. García⁸, Eric S. Menges⁹, Pedro F. Quintana-Ascencio¹⁰, Hal Caswell^{2,3,11,12}, Annette Baudisch³ & James W. Vaupel^{1,3,13}



Traditional measures capture this diversity

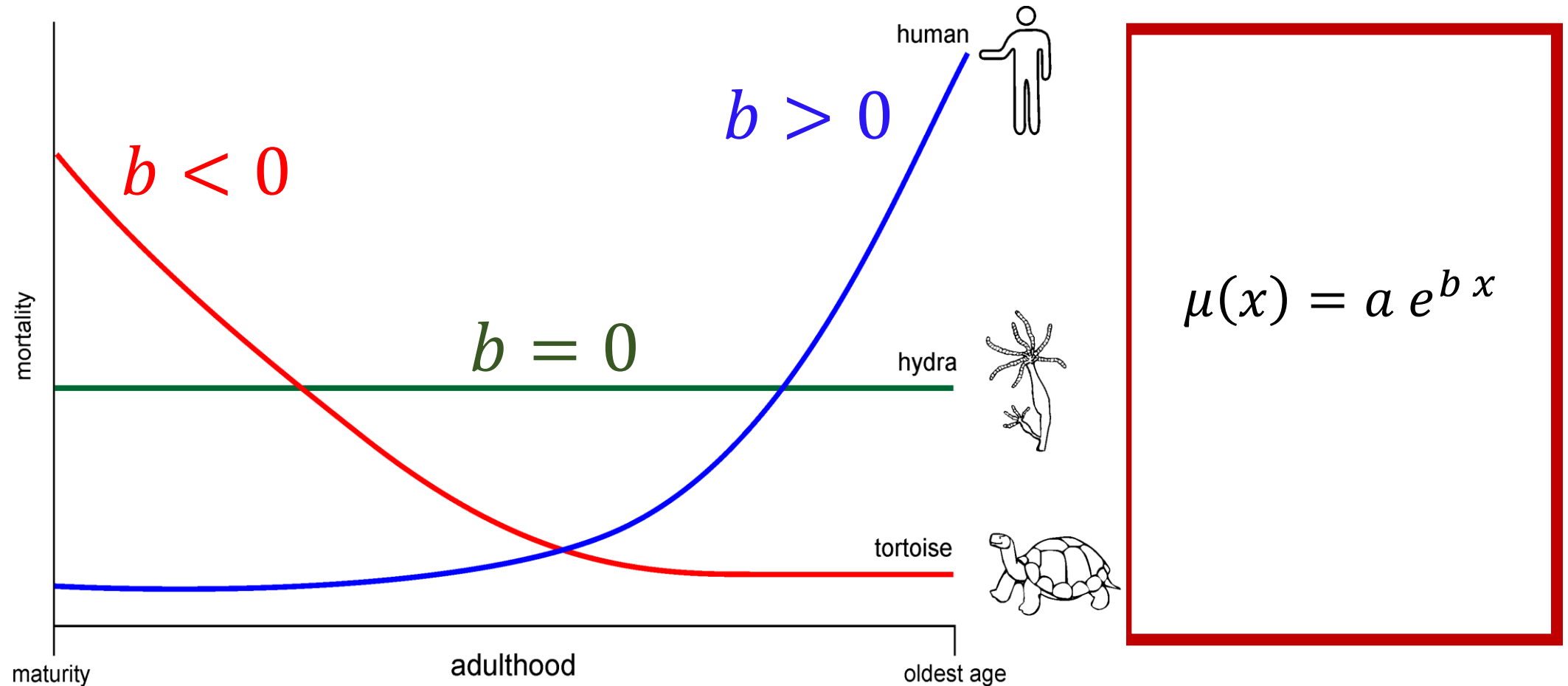


Fig. adapted from Baudisch & Vaupel 2012 Science

But...

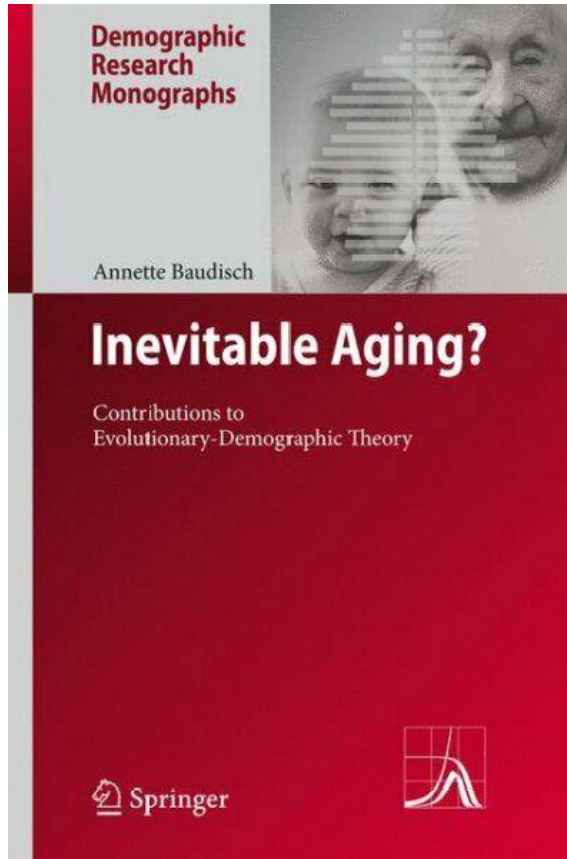
Is it enough to measure the initial mortality rate and the rate of aging to classify aging patterns?

Traditional measures

- well justified (intuitive, widely applicable, flexible, simple)
- but do they capture all relevant dimensions?

Evolutionary demographic theory ...

Theoretical diversity of patterns



Open Access Online Available
Baudisch 2008 Springer

Evolutionary demographic models
Vaupel et al 2004
Baudisch 2008
Baudisch & Vaupel 2010

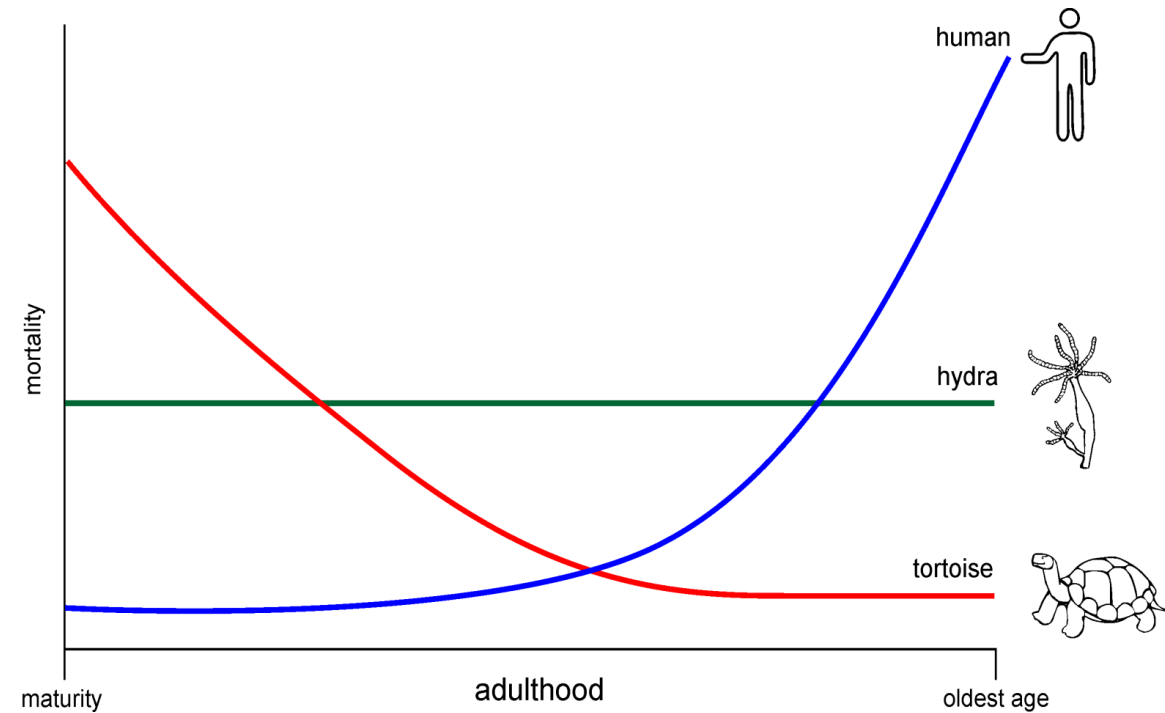
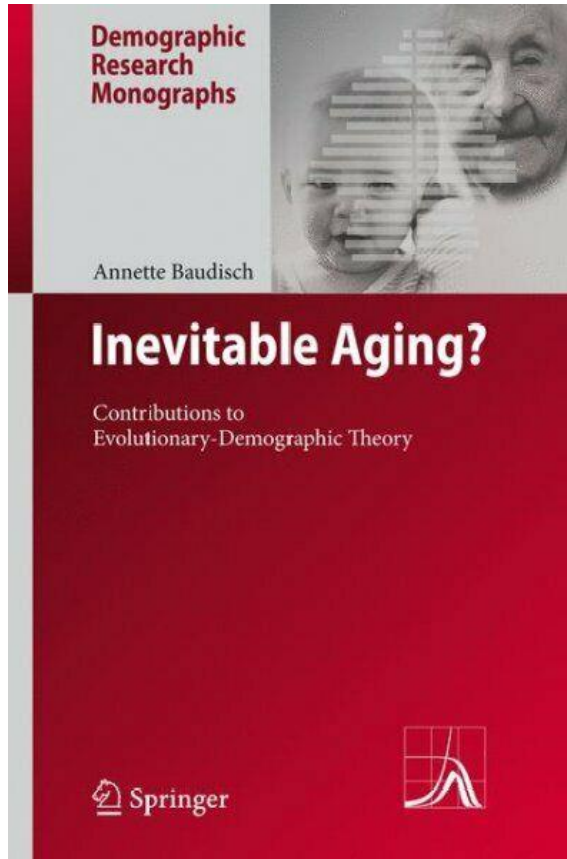


Fig. adapted from Baudisch & Vaupel 2012 Science

Theoretical diversity of patterns



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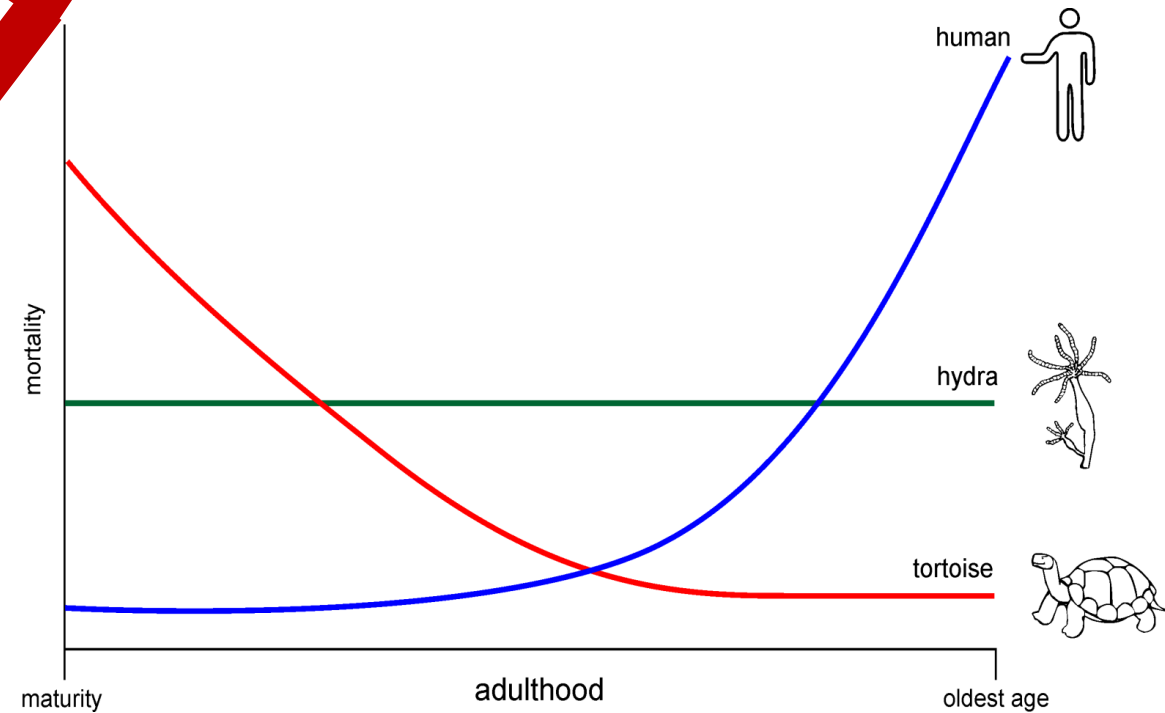
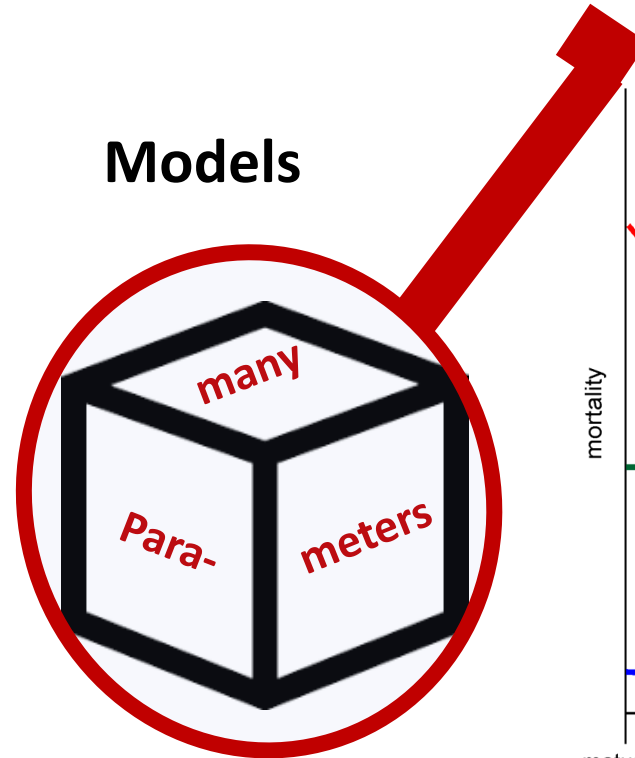
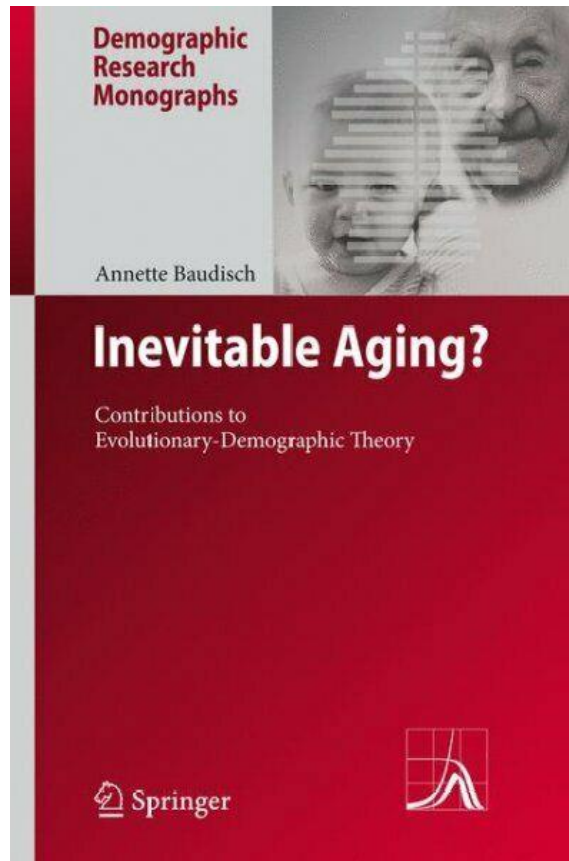
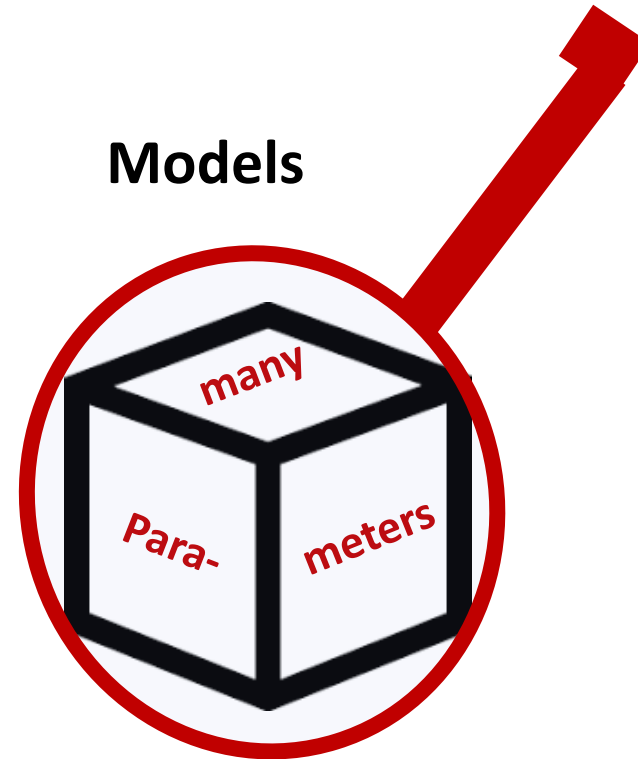


Fig. adapted from Baudisch & Vaupel 2012 Science

Theoretical diversity of patterns

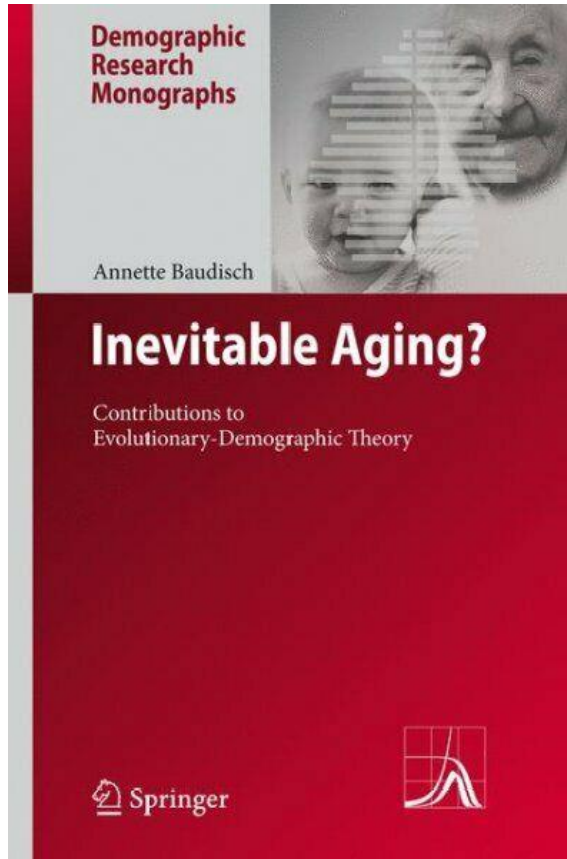


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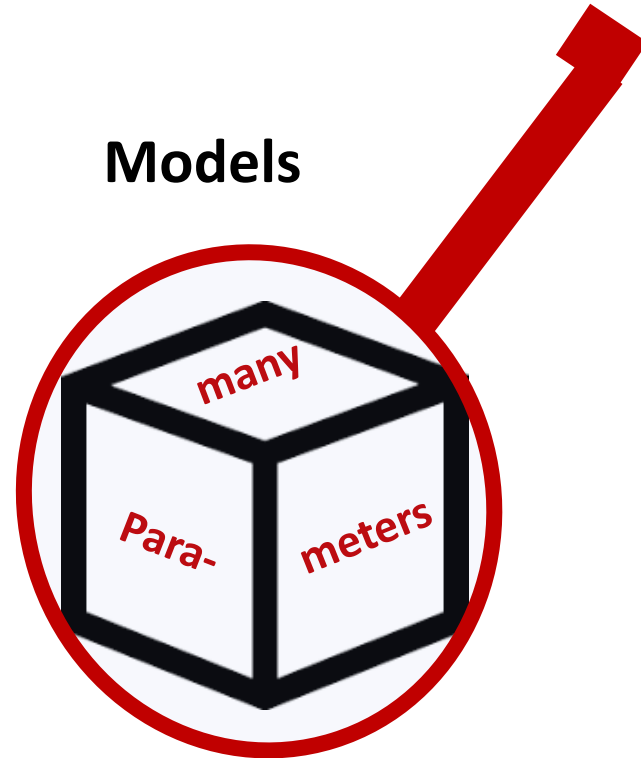


- Which parameters are important?
- What do they tell us?

Theoretical diversity of patterns

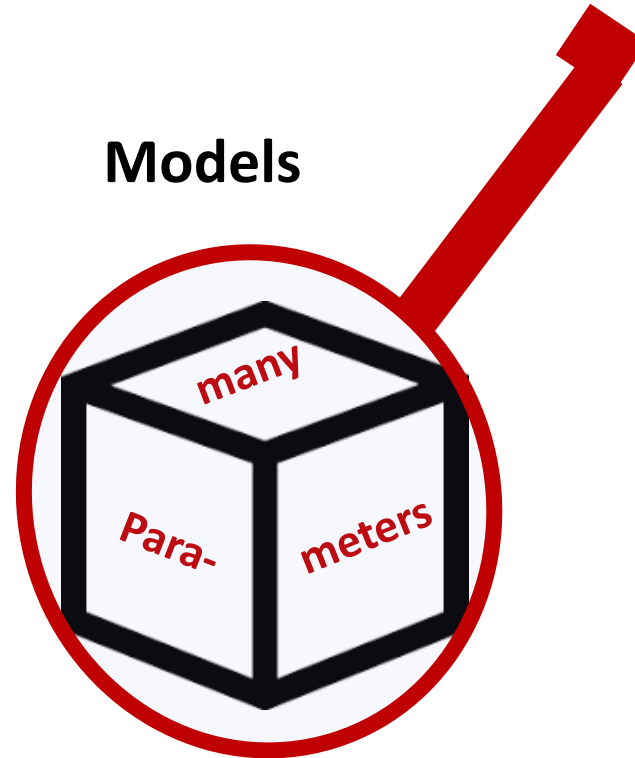
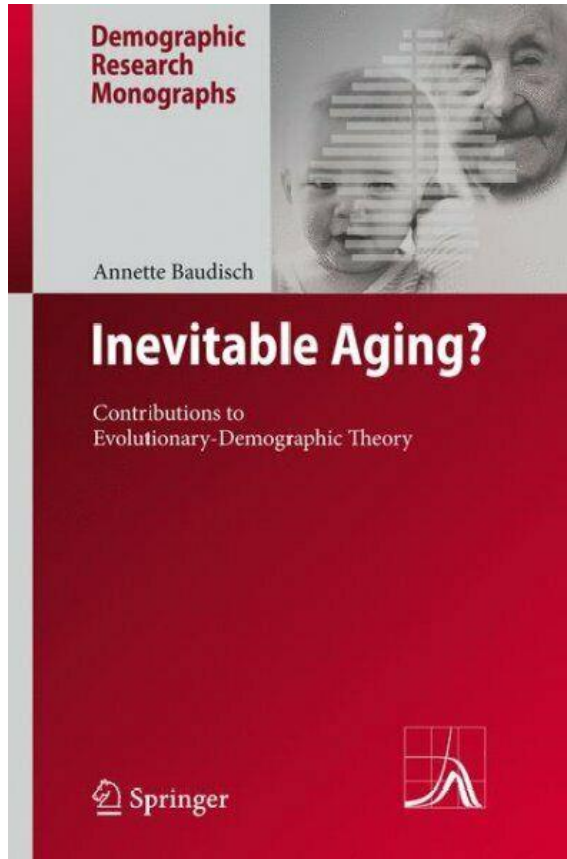


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Baudisch 2008 Springer



→ Parameters that determine what type of aging pattern evolves are **dimensionless!**

Theoretical diversity of patterns



To distinguish
different
types of aging
one needs to
factor out time.

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Baudisch 2008 Springer



To reveal a new and relevant signal of what determines aging patterns

→ Distinguish two separate dimensions

Two Dimensions of Aging

Pace of mortality

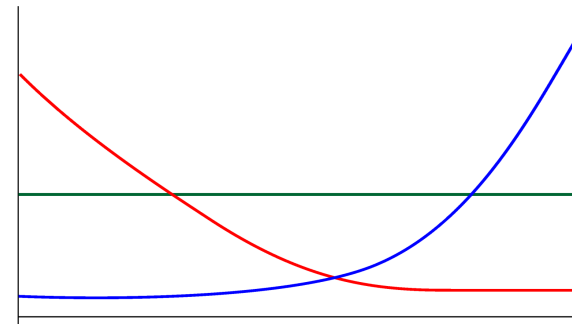
Time scale of life



To live how many more or less days, weeks, months, years, or centuries?

Shape of mortality

Time-standardized pattern of mortality over age



To experience mild or strong change in mortality over the life-course, and in what direction?

A framework motivated by Theory

Pace of mortality

Time scale of life

- Lifespan
- Death rate



Preferable Measure

*Life-expectancy, e_0 ,
at initial age 0*

$$e_0 = \int_0^{\omega} x f(x) dx$$

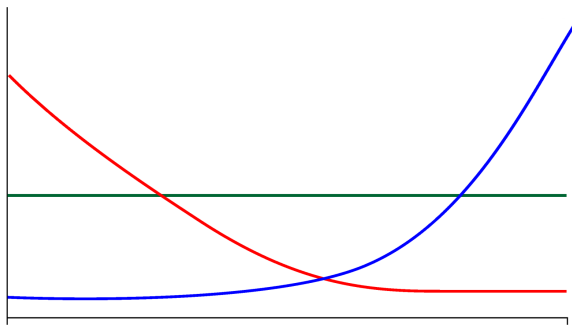
*Fraction dying
at age x*

Wrycza & Baudisch
(2014) *The Pace of Aging.*
Demographic Research

A framework motivated by Theory

Shape of mortality

Time-standardized
pattern over the life
course



Preferable Measures

Measures of
relative inequality
in ages at death

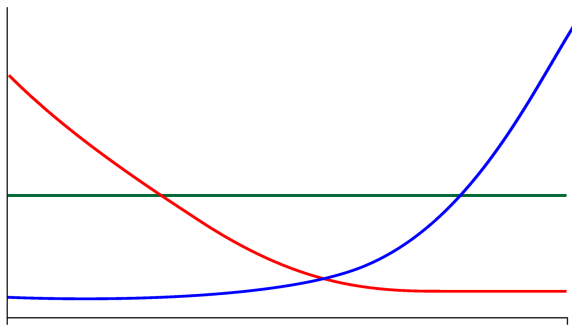
- Gini coefficient
- Life table entropy
- Coeff. of Variation

Wrycza, Missov & Baudisch (2016)
Quantifying the shape of aging.
Demographic Research

A framework motivated by Theory

Shape of mortality

Time-standardized
pattern over the life
course



Preferable Measures

Measures of
relative inequality
in ages at death

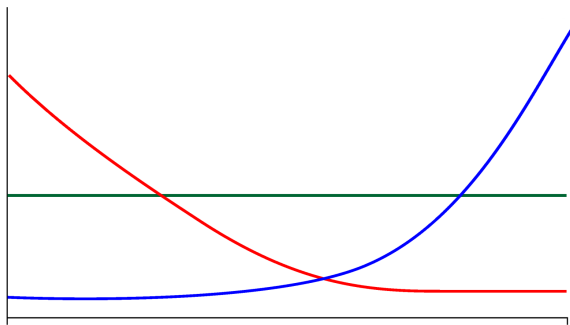
- Gini coefficient
- **Life table entropy**
- Coeff. of Variation

Wrycza, Missov & Baudisch (2016)
Quantifying the shape of aging.
Demographic Research

A framework motivated by Theory

Shape of mortality

Time-standardized
pattern over the life
course



Helpful concept

Life-years lost
due to death e^\dagger

$$e^\dagger = \int_0^{\omega} \underbrace{e(x)}_{\text{Life-years left at age } x} \underbrace{f(x)}_{\text{Fraction dying at age } x} dx$$

Vaupel and Canudas-Romo, 2003
Goldman and Lord 1986, Hakkert 1987, Vaupel 1986

A framework motivated by Theory

Shape of mortality

<i>Life-years lost</i>	$\frac{15.2}{66.1} = 0.23$	1950
$\frac{e^\dagger}{e_0}$		
<i>Life-expectancy</i>	$\frac{10.4}{78.9} = 0.13$	2010

Average values across countries
in the HMD

Helpful concept

Life-years lost due to death e^\dagger

$$e^\dagger = \int_0^{\omega} \overbrace{e(x)}^{\text{Life-years left at age } x} \underbrace{f(x)}_{\text{Fraction dying at age } x} dx$$

Vaupel and Canudas-Romo, 2003
Goldman and Lord 1986, Hakkert 1987, Vaupel 1986

A framework motivated by Theory

Shape of mortality

A measure of
relative lifespan inequality

$$\mathcal{H} = \frac{e^\dagger}{e_0}$$

Lifetable Entropy

Leser 1955, Demetrius 1974, Keyfitz 1977

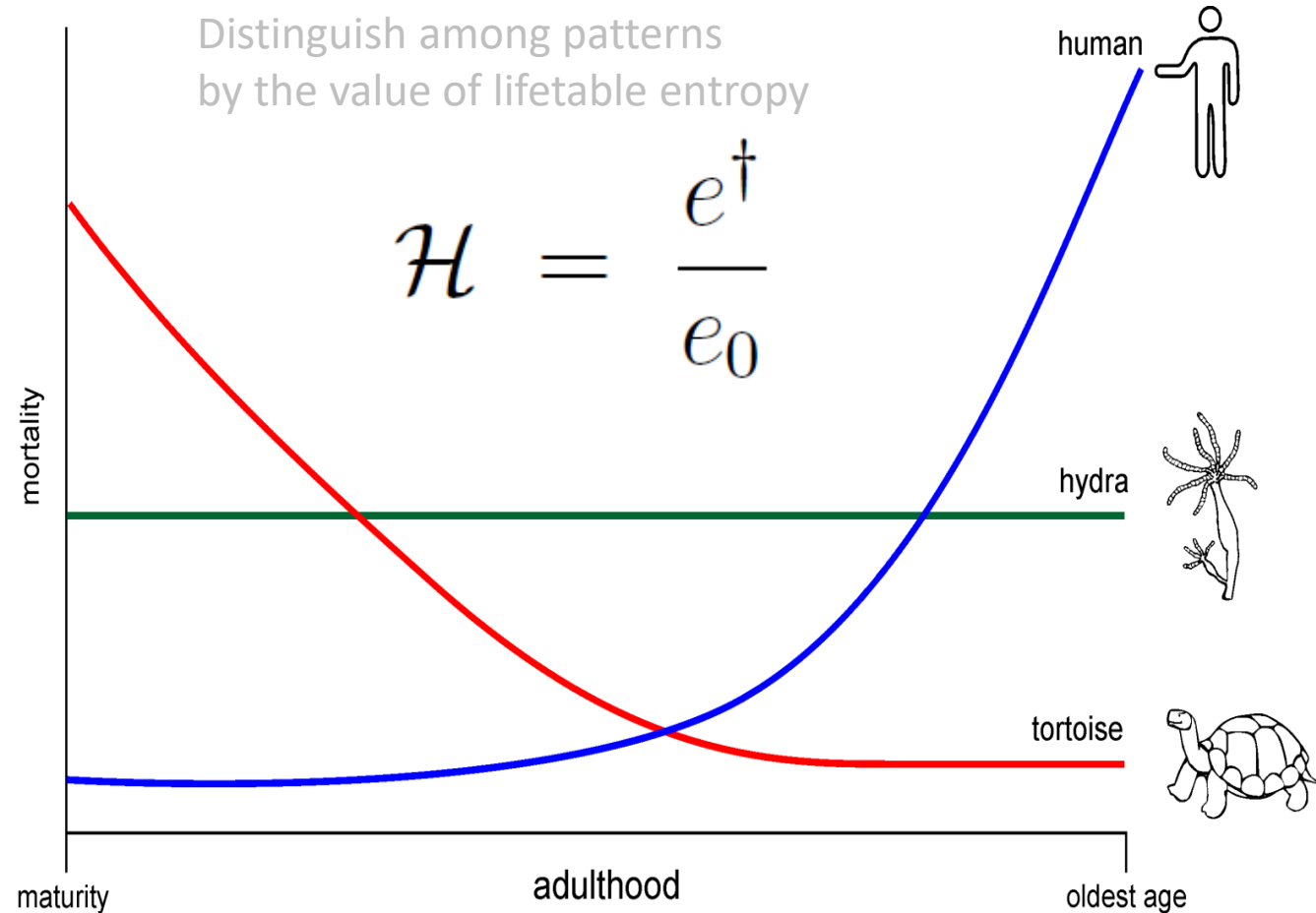
Helpful concept

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Vaupel and Canudas-Romo, 2003
Goldman and Lord 1986, Hakkert 1987, Vaupel 1986

Lifespan disparity measures capture shape of aging



$$\mathcal{H} < 1 \quad \text{if } e^\dagger < e_0$$

$$\mathcal{H} = 1 \quad \text{if } e^\dagger = e_0$$

$$\mathcal{H} > 1 \quad \text{if } e^\dagger > e_0$$

An elegant relationship

$$a e_0 + b e^{\dagger} = 1$$

Lifespan

Lifespan Disparity

Level of Mortality

Rate of Aging

The diagram shows the equation $a e_0 + b e^{\dagger} = 1$. Four green arrows point from descriptive labels to the variables in the equation: 'Lifespan' points to e_0 , 'Lifespan Disparity' points to e^{\dagger} , 'Level of Mortality' points to a , and 'Rate of Aging' points to b .

Life expectancy at birth

$$e_0 = \int_0^{\omega} \ell(x) dx$$

Preston, Heuveline, Guillot 2000

Life years lost at death

$$e^{\dagger} = -\int_0^{\omega} \ell(x) \ln \ell(x) dx$$

Vaupel and Canudas-Romo, 2003
Goldman and Lord 1986, Hakkert 1987, Vaupel 1986

Gompertz Mortality

$$\mu(x) = a e^{bx}$$

Gompertz (1825)

An elegant relationship

Why does it matter?

$$a e_0 + b e^\dagger = 1$$

How can it be helpful?

Life expectancy at birth

$$e_0 = \int_0^\omega \ell(x) dx$$

Preston, Heuveline, Guillot 2000

Life years lost at death

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Vaupel and Canudas-Romo, 2003

Goldman and Lord 1986, Hakkert 1987, Vaupel 1986

Gompertz Mortality

$$\mu(x) = a e^{bx}$$

Gompertz (1825)

An elegant relationship

It relates two key demographic summary measures

$$a e_0 + b e^{\dagger} = 1$$

for a general mortality model, which broadly captures not just human, but also non-human mortality patterns across adult ages (Finch et al 1990).

Life expectancy at birth

$$e_0 = \int_0^{\omega} \ell(x) dx$$

Preston, Heuveline, Guillot 2000

Life years lost at death

$$e^{\dagger} = -\int_0^{\omega} \ell(x) \ln \ell(x) dx$$

Vaupel and Canudas-Romo, 2003
Goldman and Lord 1986, Hakkert 1987, Vaupel 1986

Gompertz Mortality

$$\mu(x) = a e^{bx} \quad \text{Gompertz (1825)}$$

An elegant relationship

It aids formal demographic analysis

$$a e_0 + b e^{\dagger} = 1$$

and supports increasing interest in the relationship of lifespan and lifespan inequality in particular.

For example:

Life expectancy at birth

$$e_0 = \int_0^{\omega} \ell(x) dx$$

Preston, Heuveline, Guillot 2000

Life years lost at death

$$e^{\dagger} = -\int_0^{\omega} \ell(x) \ln \ell(x) dx$$

Vaupel and Canudas-Romo, 2003
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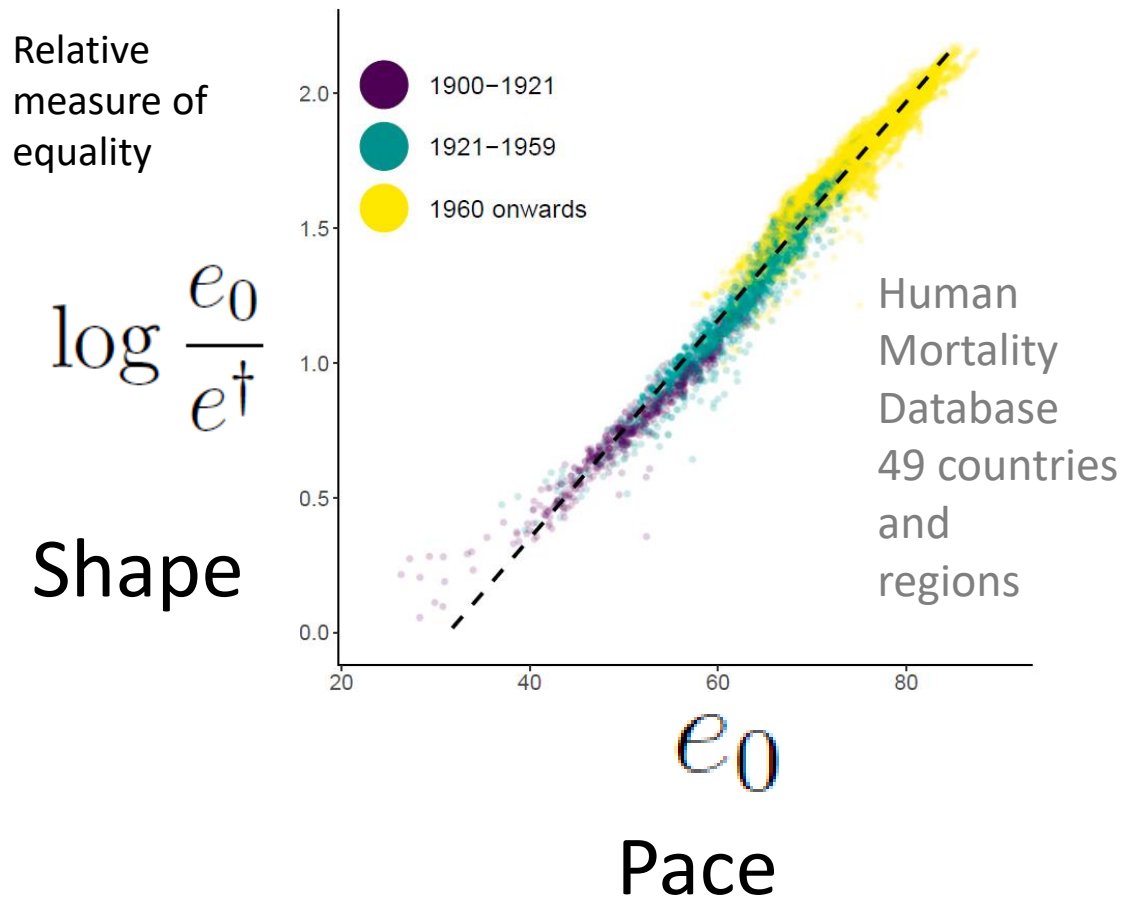
Gompertz Mortality

$$\mu(x) = a e^{bx} \quad \text{Gompertz (1825)}$$

Dynamics of life expectancy and life span equality

Fig 1 Aburto et al. 2019

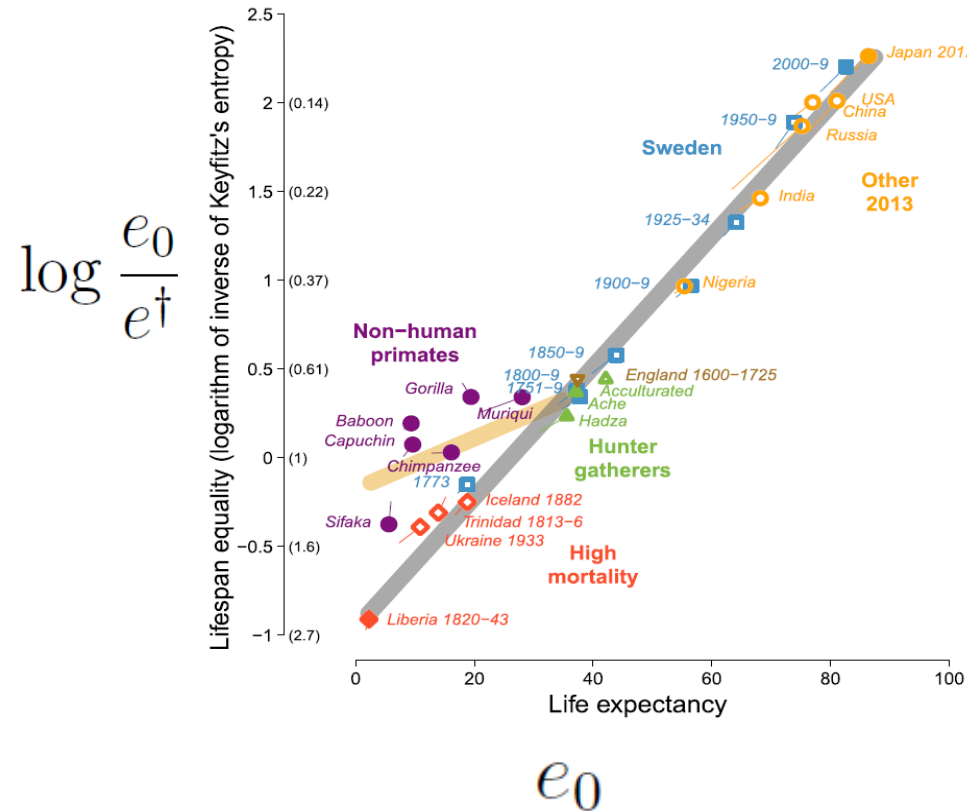
Fig 1 Aburto et al. 2019



Aburto et al. “*develop a unifying framework to study life expectancy and life span equality over time, relying on concepts about the pace and shape of aging*”

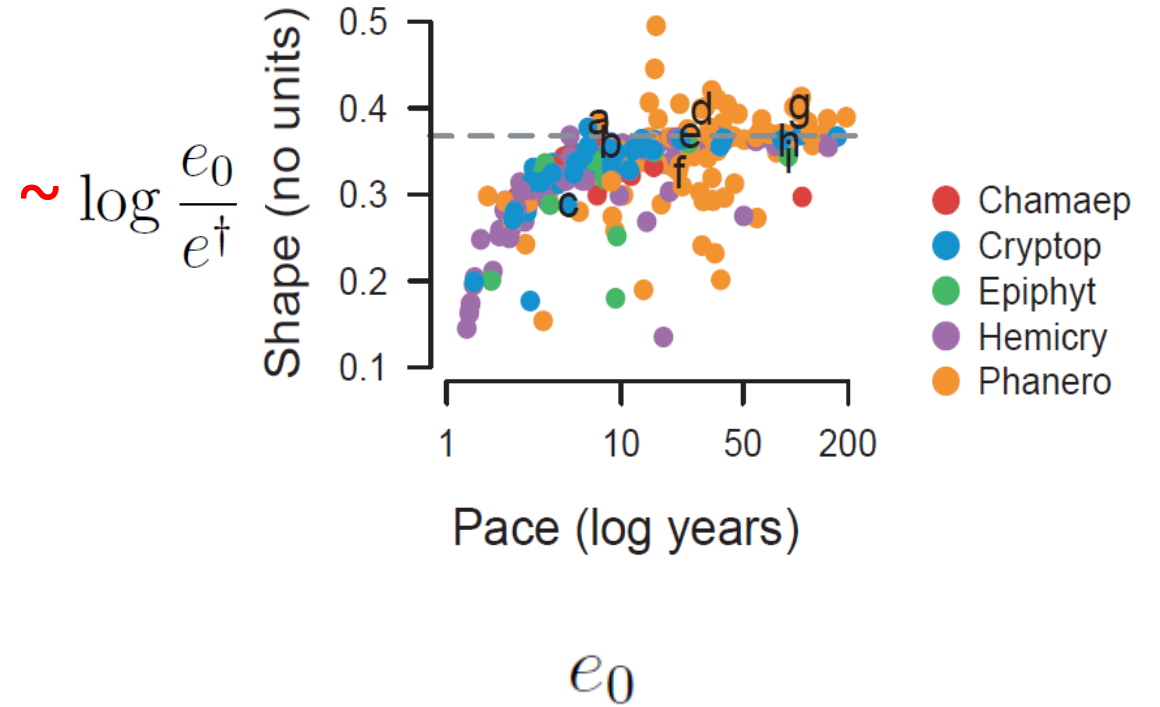
Lifespan (Pace) vs Lifespan Variation (Shape)

People (and other primates)



Colchero et al 2016, PNAS

Plants (with flowers)



Baudisch et al 2013, JEcology

An elegant relationship

Support comparative research across the tree of life on lifespan (pace) and lifespan variation (shape)

$$a e_0 + b e^{\dagger} = 1$$

Supports to explore dynamics in pace shape space

Life expectancy at birth

$$e_0 = \int_0^{\omega} \ell(x) dx$$

Preston, Heuveline, Guillot 2000

Life years lost at death

$$e^{\dagger} = -\int_0^{\omega} \ell(x) \ln \ell(x) dx$$

Vaupel and Canudas-Romo, 2003
Goldman and Lord 1986, Hakkert 1987, Vaupel 1986

Gompertz Mortality

$$\mu(x) = a e^{bx} \quad \text{Gompertz (1825)}$$

Further relationships

$$ae'_{0a} + be'_{0b} = 1$$

$$ae'_{\dagger a} + be'_{\dagger b} = 1$$

$$a\mathcal{H}_a + b\mathcal{H}_b = 0$$

Supports exploring dynamics in pace shape space

Notation
Relative
Change:

$$\dot{x}_a = \frac{\frac{dx}{da}}{x}$$

Contributes to a large body of research

e.g.,

Wilmoth, Horiuchi 1999, Edwards,
Tuljapurkar 2005, Smits, Monden 2009,
Tuljapurkar 2010, Edwards 2011, Baudisch
2011, van Raalte, Caswell 2013, Fernandez,
Beltrán-Sánchez 2015, Vaupel et al. 2015,
Colchero et al. 2016, van Raalte et al. 2018,
Ebeling et al. 2018, Permanyer, Scholl 2019,
Aburto et al. 2019, Vaupel et al. 2021

**Increasing interest
in lifespan inequality
and its relationship
with lifespan**

Extended the framework to fertility

(2019) A pace and shape perspective on fertility
Baudisch & Stott. *Methods in Ecology and Evolution*



Born once.



Die once.

(2021) Born once, die once: Life table relationships for fertility
Baudisch & Alvarez. *Demographic Research*

ERC Consolidator grant 2022.

Towards a Transdisciplinary Theory of Birth and Death



Born once.



Die once.

Using the tools of formal demography to conceptualize and model birth and death of "individuals", eventually across disciplines.

Summary & Conclusion

Population aging, species extinction, sustainability crisis, pandemics,... all major problems of the world result from an imbalance of birth and death processes.

Formal Demography is the key to conceptualize and model birth and death processes (+ migration!) in whatever disciplines.

The world needs more formal demographers!

Comments Questions

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