

# Person-years lost from Covid

BFDW, Day 2, Lecture 3

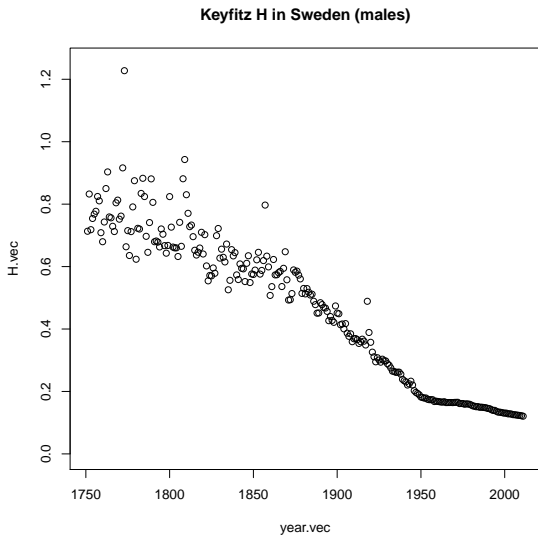
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# Our plan

- ▶ Life expectancy criticisms
- ▶ Remaining person years lost as an alternative
- ▶ Mathematics of lost lives and lost life
- ▶ Elegance  $\neq$  relevance?

# Technical problem: $\mathcal{H}$ is not constant



Populations with different base mortality will see different changes

## Substantive problem: isn't $e_0$ a “misleading indicator”?

*In the context of epidemic mortality, life expectancy at birth is a misleading indicator, because it implicitly assumes the epidemic is experienced each year over and over again as a person gets older.*

– Goldstein and Lee (2000)

## $e_0$ as “standardization”

Life expectancy is reciprocal of standardized mortality, with period survivorship as standard.

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## $e_0$ as “standardization”

Life expectancy is reciprocal of standardized mortality, with period survivorship as standard.

$$\frac{\int h(a)\ell(a) da}{\int \ell(a) da} = \frac{1}{e_0}$$

So with different life tables, we'll have different standard pop  $\ell(x)$ .

## Loss of person years

- ▶ The members of a population each have some expected future years of life.
- ▶ When a crisis kills people, a portion of that future life is lost.
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## Proportion of remaining life lost

Our equation (3) from “cheatsheet”

$$\frac{\Delta\theta_0}{\theta_0} \approx -\frac{\mathcal{H}}{A_0}\delta,$$

where  $\theta_0$  is the number of remaining person-years in the population before the crisis.

## Set up

Assume a stationary population and proportional crisis

- ▶ Person-years remaining before crisis

$$\int N(a)e(a) da = \int Bl(a)e(a) da$$

- ▶ Person-years lost

$$\int D(a)e(a) da = \int Bl(a)\delta h(a)e(a) da$$

Combining, proportion of remaining person years of life:

$$\frac{\Delta\theta_0}{\theta_0} = \frac{\text{PY lost}}{\text{PY remaining}} = -\delta \frac{\int \ell(a)h(a)e(a) da}{\int \ell(a)e(a) da}$$

# Evaluating

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Dividing top and bottom by  $e_0$  gives

$$\frac{\Delta\theta_0}{\theta_0} = \frac{\text{PY lost}}{\text{PY remaining}} = -\delta \frac{\mathcal{H}}{A_0}$$

## An example

Say  $\delta = 1/2$ ,  $H = 0.15$ ,  $A_0 = 40$

Then,

$$\frac{\Delta\theta_0}{\theta_0} = \frac{\text{PY lost}}{\text{PY remaining}} = -\delta \frac{\mathcal{H}}{A_0} =$$

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Why so small, when mortality increased by so much?

$$\frac{\text{PY Lost}}{\text{PY Remaining}} = \frac{D_{\text{crisis}} \cdot \bar{e}(\text{dying})}{N \cdot \bar{e}(\text{living})}$$

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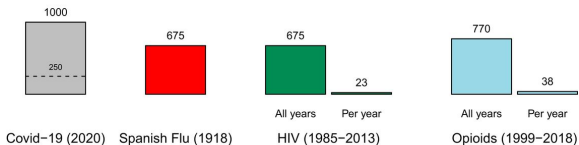
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So we have 1/4 of a per-capita death rate . . . , a very small number. (Mostly, because the base rate of mortality is already small.)

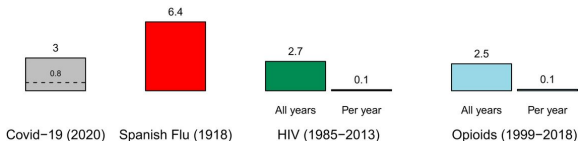


# Comparing to baseline (Goldstein and Lee, 2020)

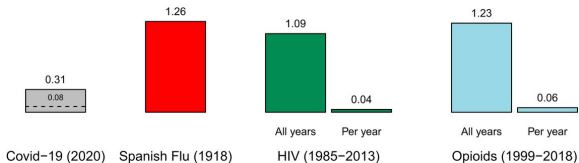
## Epidemic deaths (in thousands)



## Epidemic deaths / Population size (per thousand)



## Life years lost, relative to non-epidemic mortality



# Breakout Exercises

Our usual A, B, C (but spiced up with some controversy?)

- A Calculate person years in population
- B Calculate person years lost
- C Compare to our approximation

# Discussion

Your questions first

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## Your questions first

- ▶ About how many person years were lost per person in the US from Covid in 2020? Multiple years, multiple weeks, ...
- ▶ Who's right: Ilya or Josh? (Or neither?)
- ▶ How do we think about effect on cohort life expectancy?
- ▶ Why did Spanish Flu, HIV, and Opioids result in a larger loss of remaining life?
- ▶ Should we adjust for age structure?

# Bringing it all back home (Bob Dylan just turned 80!)

## Some common threads

- ▶ Each measure ( $CDR$ ,  $e_0$ ,  $PYR$ ) tried to accomplish something
- ▶ Formal analysis simplified and identified key properties – and potential problems.
- ▶ New problems, new formulations  
Results discovered 100 years ago are still important today.

## A concluding quote

*Formal demography*

*“is nothing more than **clear analytic thinking** about a demographic problem, with hard-edged concepts, typically distilled into mathematical expressions.”*

– Ron Lee (2014),