Bayesian indirect estimation of under-five mortality from summary birth histories

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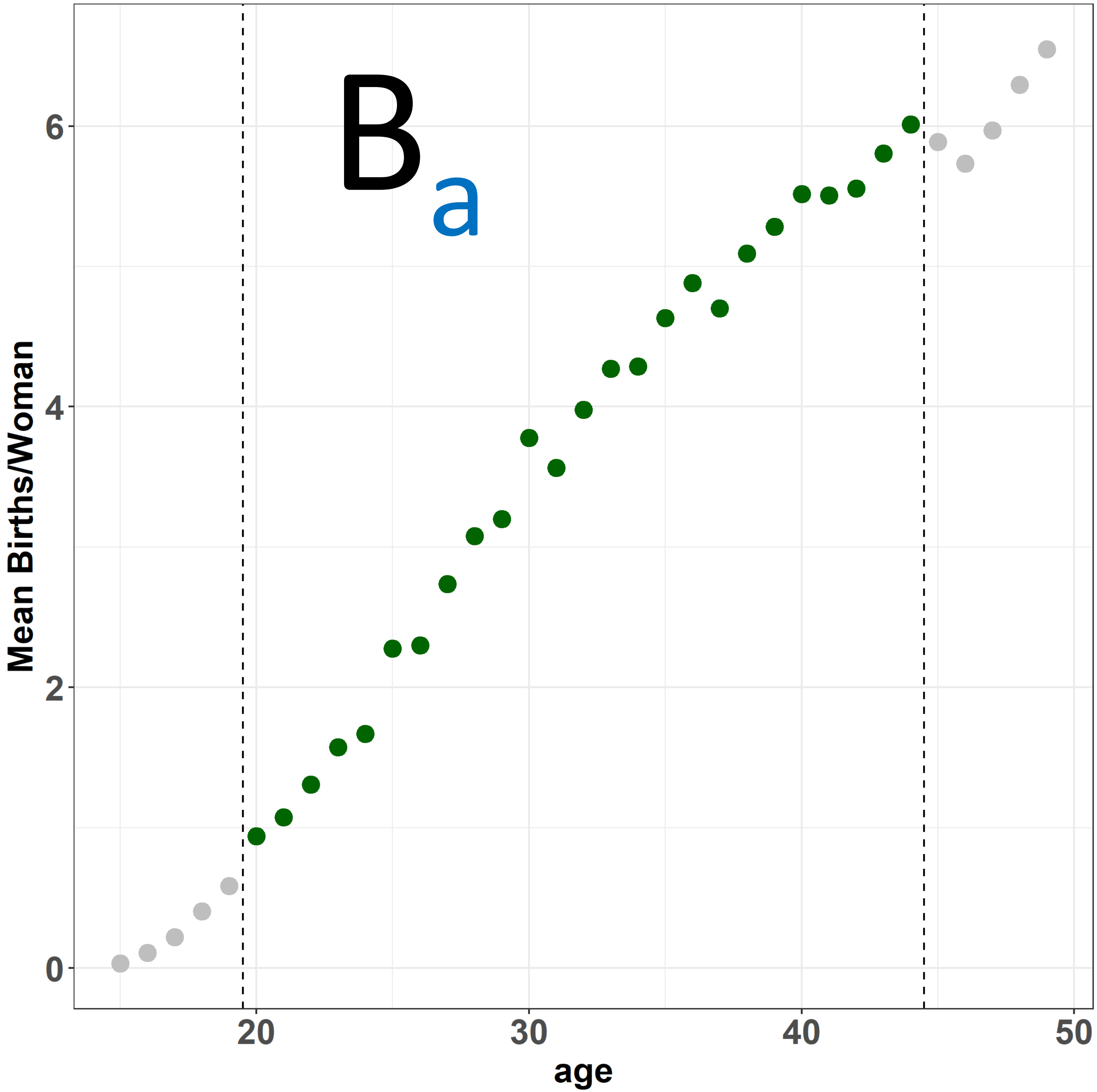
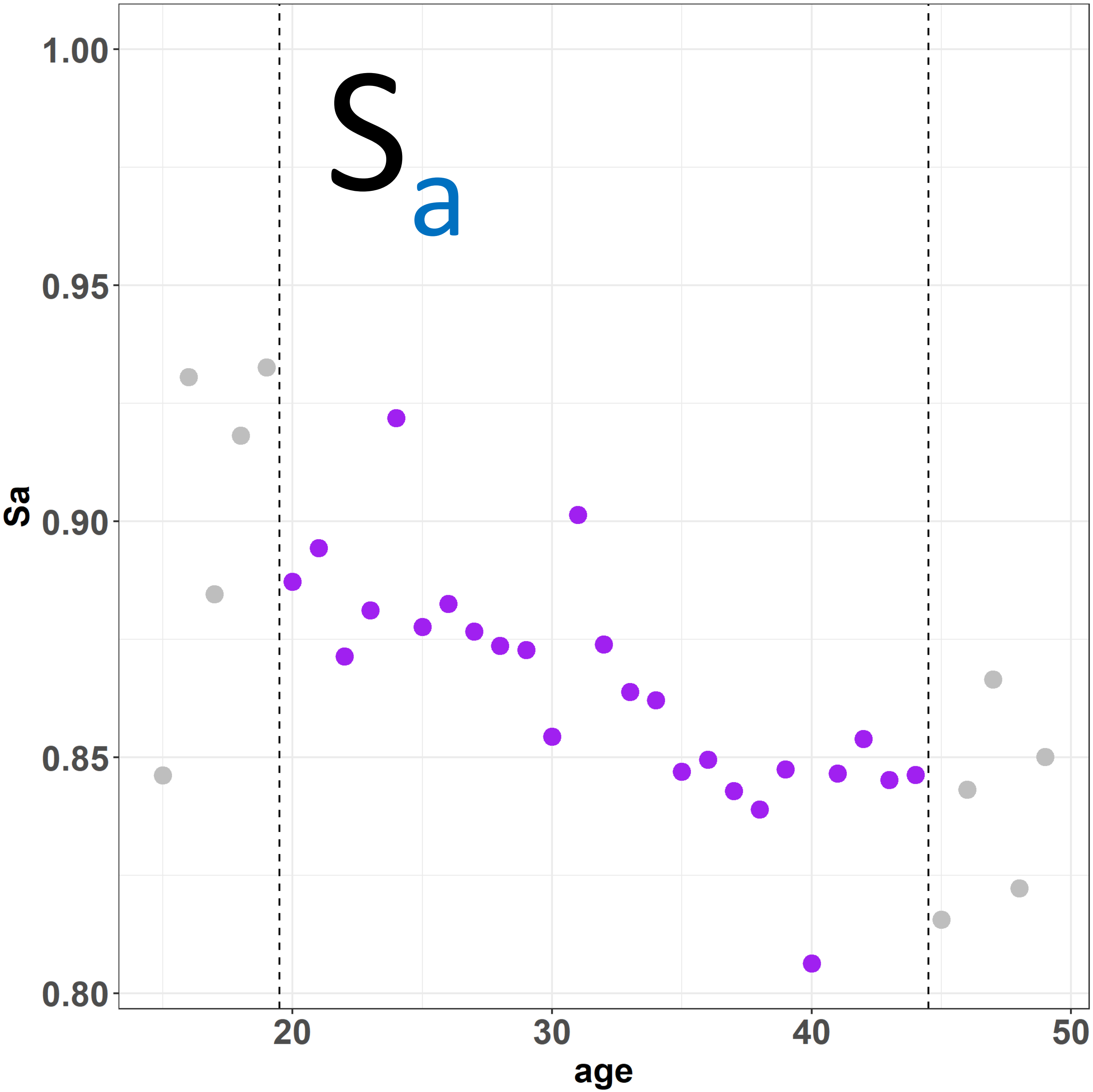
Summary Birth Histories

- Survey/census data with
  - Women's ages a
  - # children ever born $\mathbf{B}_a$ (no details on when born)
  - # children died $\mathbf{D}_a$ (no details on when died)

- Fractions surviving $\mathbf{S}_a = \mathbf{1- (D}_a \mathbf{/ B}_a\mathbf{)}$ by woman's age depend on …
  - Mortality level
  - Mortality pattern by age
  - Fertility Pattern by age
  - Time Trends in Rates


B
a
Mean Births/Woman
0
2
4
6
20
30
40
50
age



S
a
1.00
0.95
0.90
0.85
0.80
Sa
20
30
40
50
age

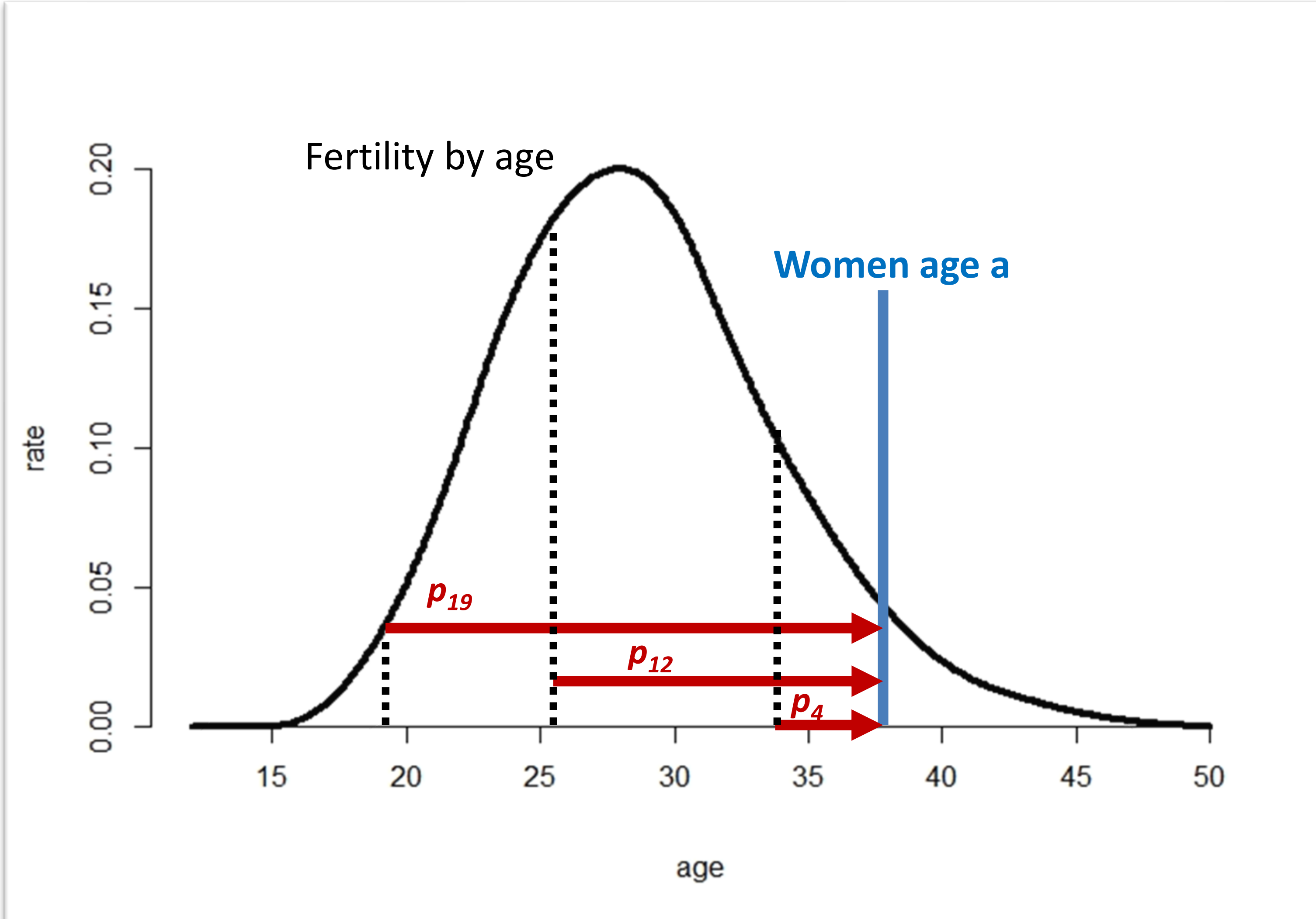
Objective

Estimate under-five mortality $q(5)$ from $\{B_a, S_a\}$

Problem

Survival of children of a-yr-olds is a mixture of survival probs for those

- born in survey year (now age $x=0$)
- born 1 year ago (now $x=1$)
- …
- born 10 years ago(now $x=10$)
- …



Expected fraction a-kids alive = Fertility-wtd avg of p_x:

$$\bar{S}_a = \frac{\sum_x f_{a-x} \, p_x}{\sum_x f_{a-x}} = \sum_x W_{ax} \, p_x$$

Brass Indirect Estimation

Brass Indirect Estimation

- Fertility patterns are robust
  - Unimodal, peak in 20s
  - Allows reasonable guesses of kids' avg mortality exposure from women's ages

- Mortality patterns are robust
  - Child mortality concentrated in infancy
  - Falling mortality rates over ages 0-5

if age-specific rates are **unchanging** → $\boldsymbol{q}(\mathbf{5}) \approx 1 - \bar{S}_{30-34}$

Feeney Time Allocation

if age-specific rates are **unchanging** → $q(5) \approx 1 - \bar{S}_{30-34}$

if fertility **constant** but mortality **changing** → $q_p(5, -5) \approx 1 - \bar{S}_{30-34}$

5 yrs ago

Remaining Problems

- Sampling noise
  (esp. if we discard data from all women ≠ 30-34)

- Uncertainty about demographic parameters
  – true age pattern of fertility
  – true age pattern of child mortality

- Changing rates
  – Fertility rates are falling rapidly in places where indirect methods are still necessary
  – Falling fertility →
    - longer times since births
    - longer exposure to mortality (higher avg **x** for women age **a**)
    - lower % of children surviving at a given level of current q(5)

A Bayesian Version of Brass

Bayesian Version: Main Ideas

- Age-specific **fertility** rates vary over time
  -> each *cohort of women* may have faced different age-specific rates in the past

a=20 in 2010: $f_{12,2002} \rightarrow f_{13,2003} \rightarrow \ldots f_{20,2010}$

a=30 in 2010: $f_{12,1992} \rightarrow f_{13,1993} \rightarrow \ldots f_{29,2009} \rightarrow f_{30,2010}$

- Age-specific **mortality** rates vary over time
  -> each *cohort of children* may have faced different age-specific survival probs in the past

x=5 in 2010: $p_{0,2005} \times p_{1,2006} \times \ldots \times p_{5,2010}$

x=10 in 2010: $p_{0,2000} \times p_{1,2001} \times \ldots \times p_{9,2009} \times p_{10,2010}$

Bayesian Version: Main Ideas

- Build parametric models for demographic rates in each period during past ≈30 yrs

- Choose priors for parameters
  [= Which sets of parameters are plausible/implausible
  before we look at any SBH data?]

- Among plausible fertility and mortality histories, find those that are also consistent with observed SBH data at women's ages a=20,21,…,44

- Summarize time trends of q(5) in the most likely histories

Fertility Model

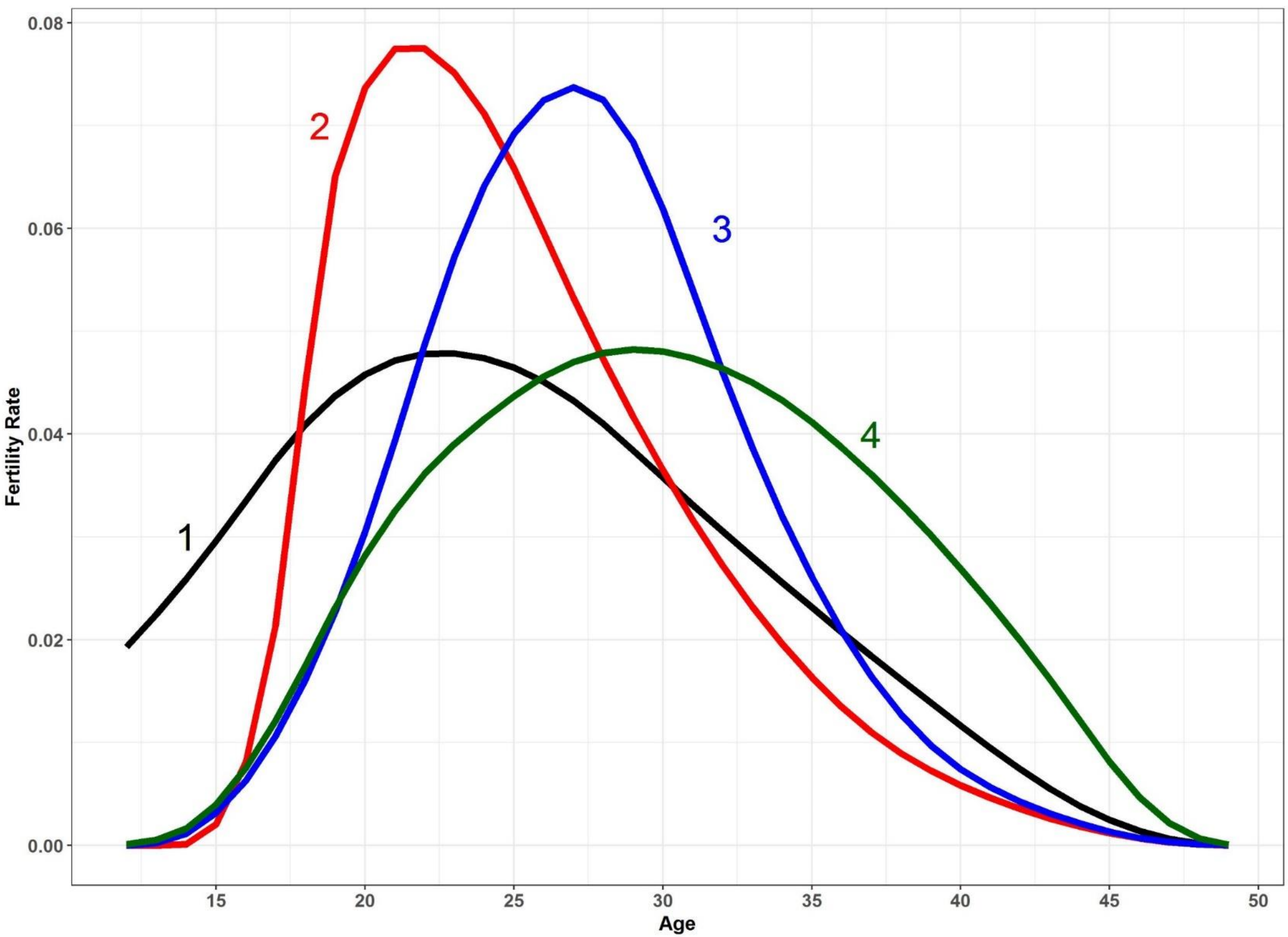
Unique rate f_{at} for each (age, period)

PARAMETERS

1. Age pattern for period *t*:
   - Weights for each of 4 *"archetypes"*
   - 1st and last period wts ~ *Dirichlet(1,1,1,1)*
   - Linear change in weights over time

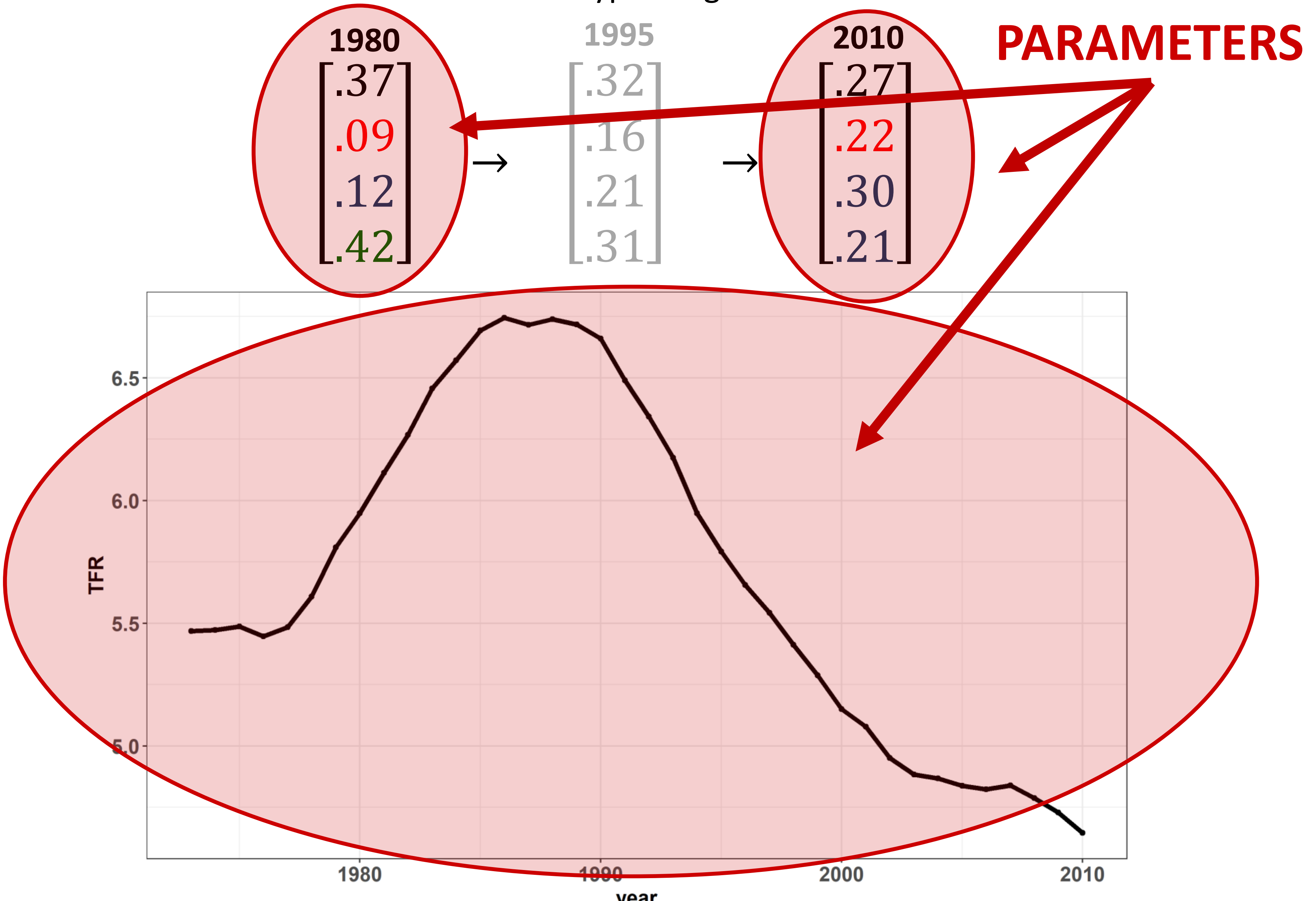
2. Level for period *t*
   - TFR_t ~ 2nd-order random walk, sd=σ_{TFR}

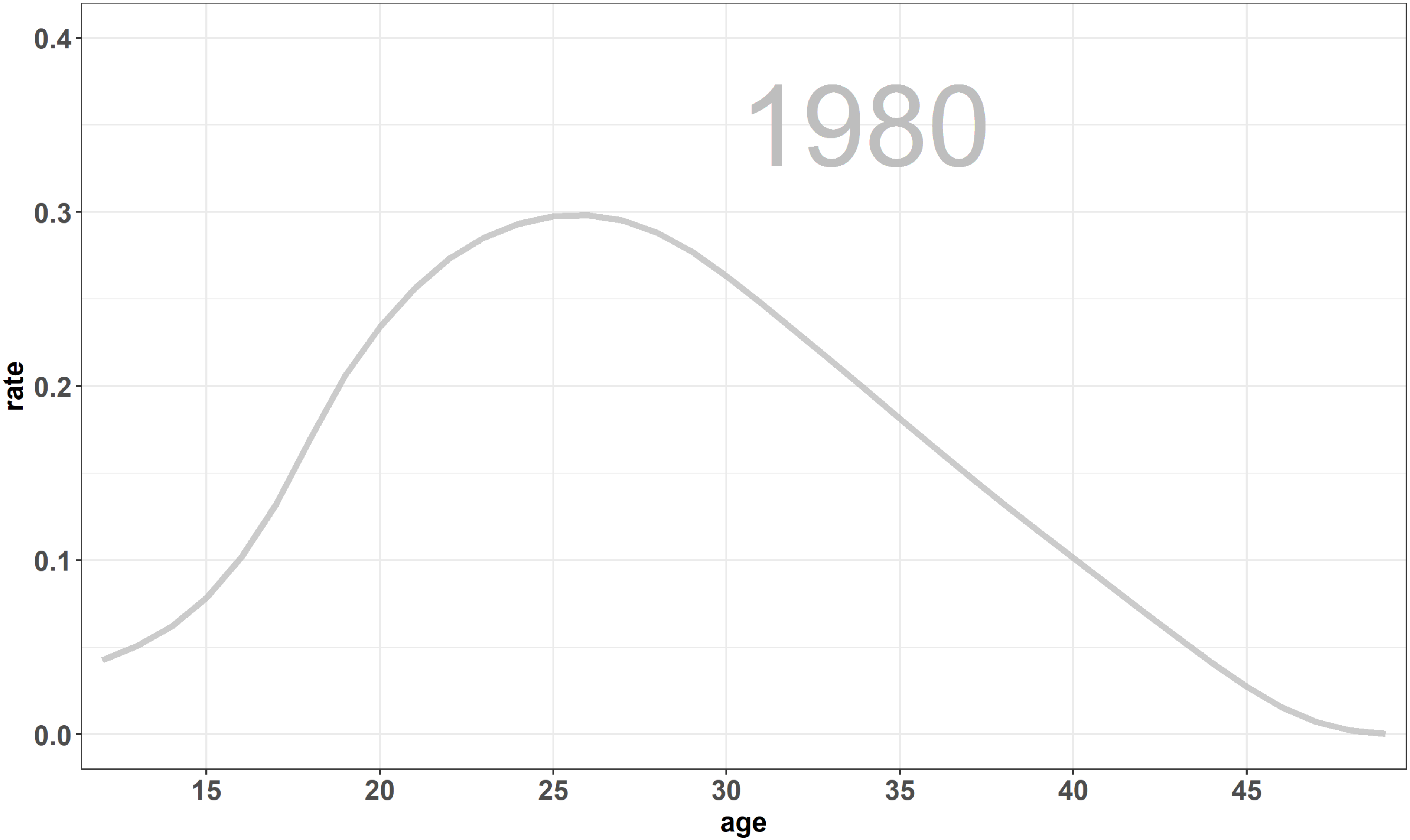
Archetypes: fertility age patterns

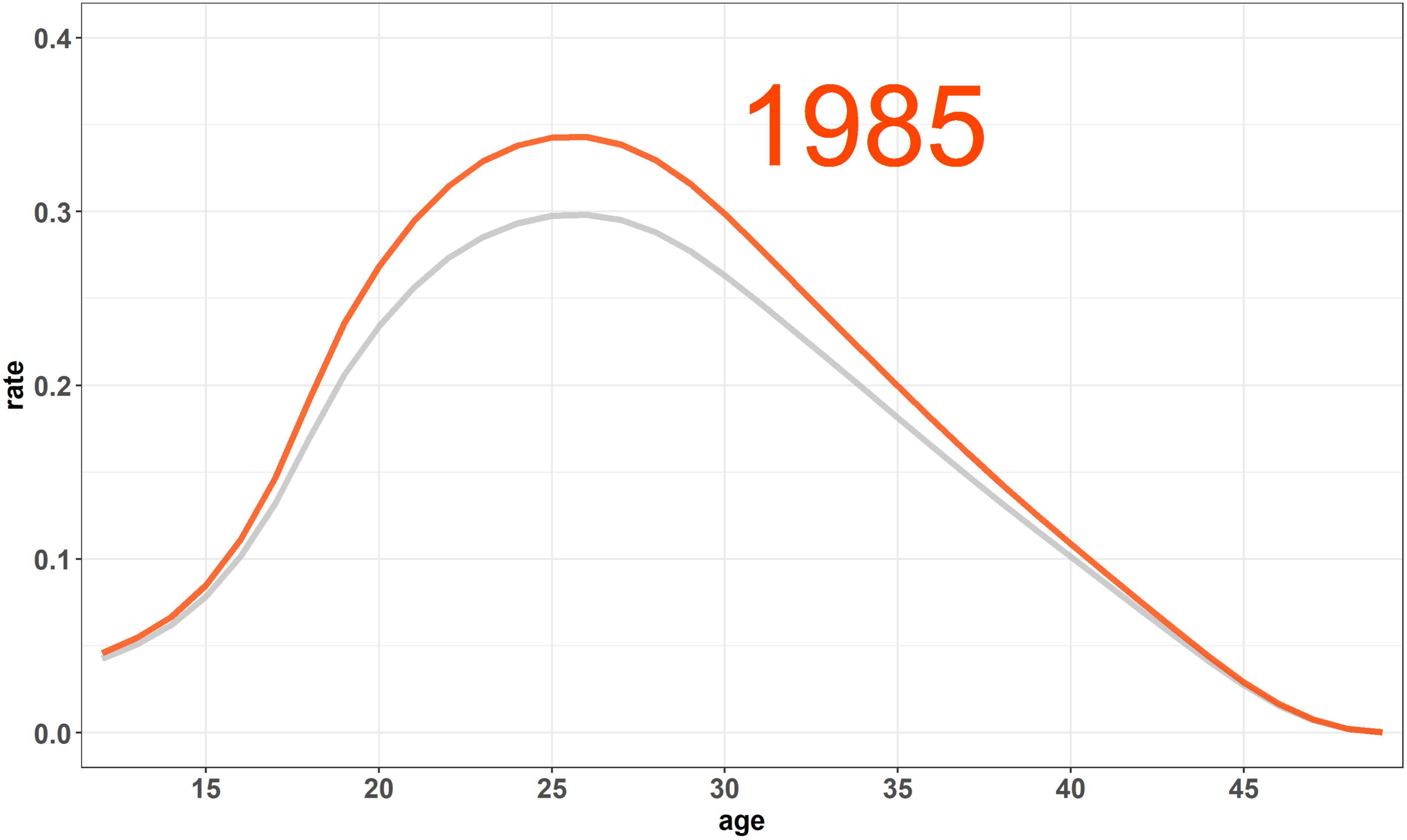


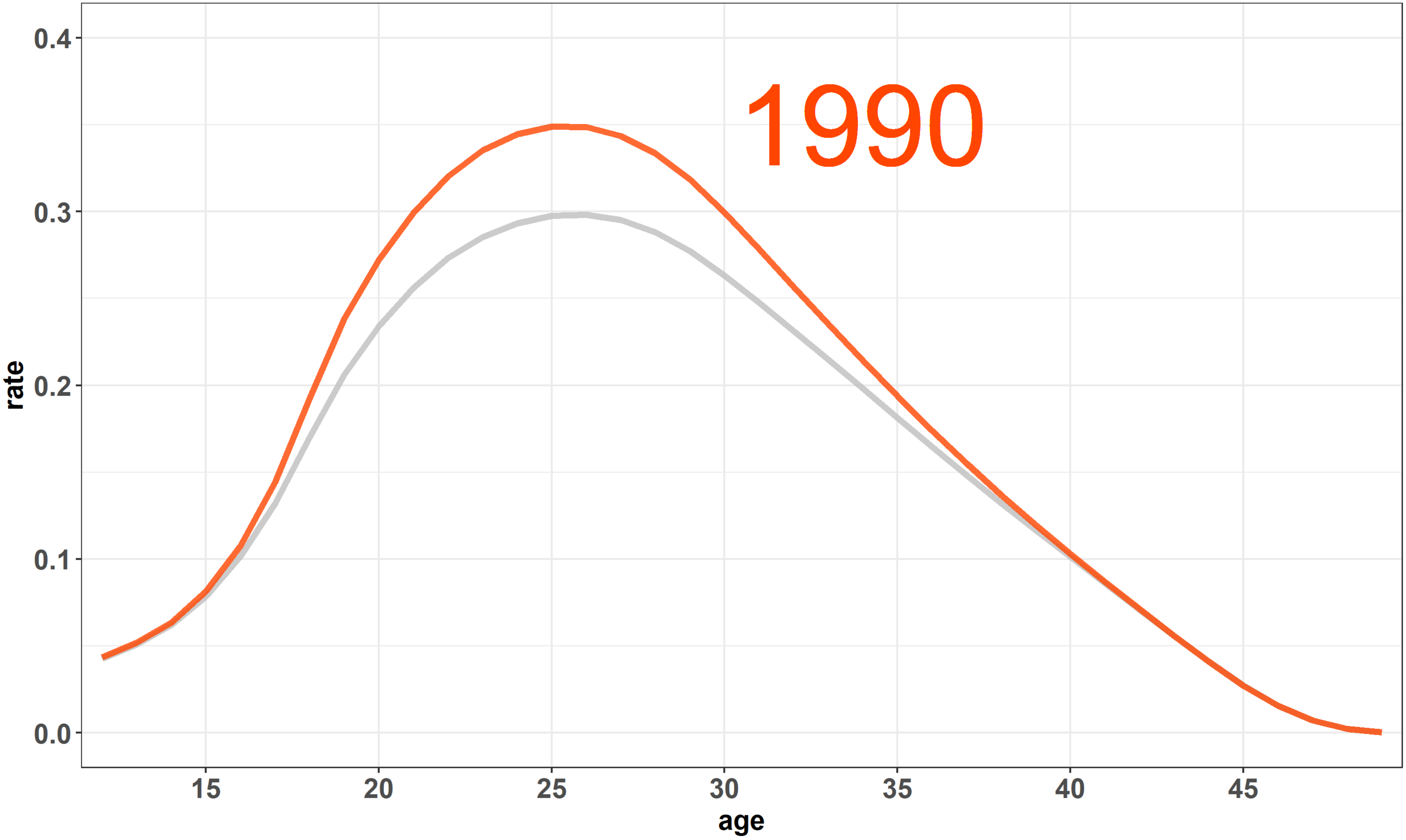
FERTILITY PARAMETERS: EXAMPLE

Archetype weights

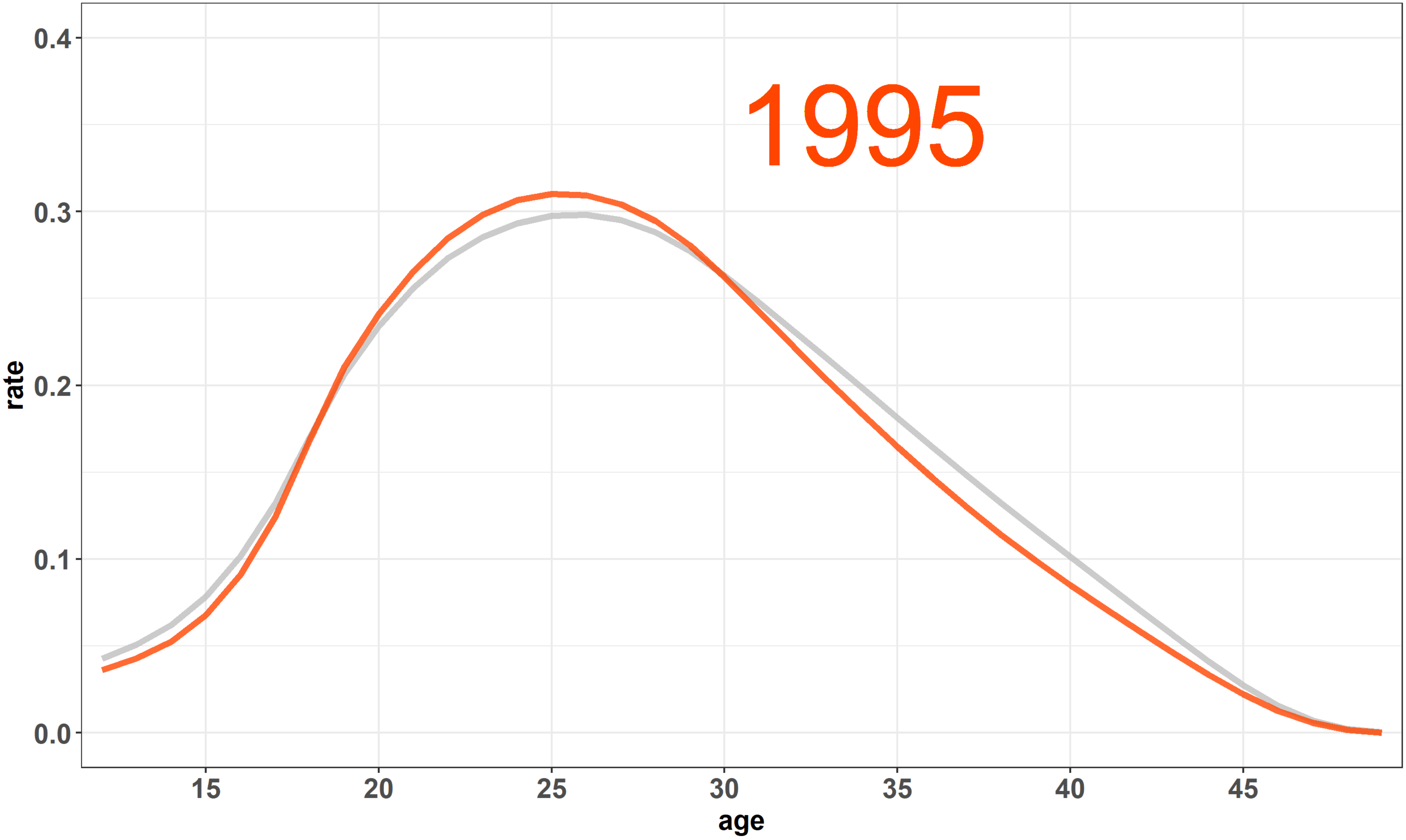



1980
rate
0.0
0.1
0.2
0.3
0.4
age
15
20
25
30
35
40
45

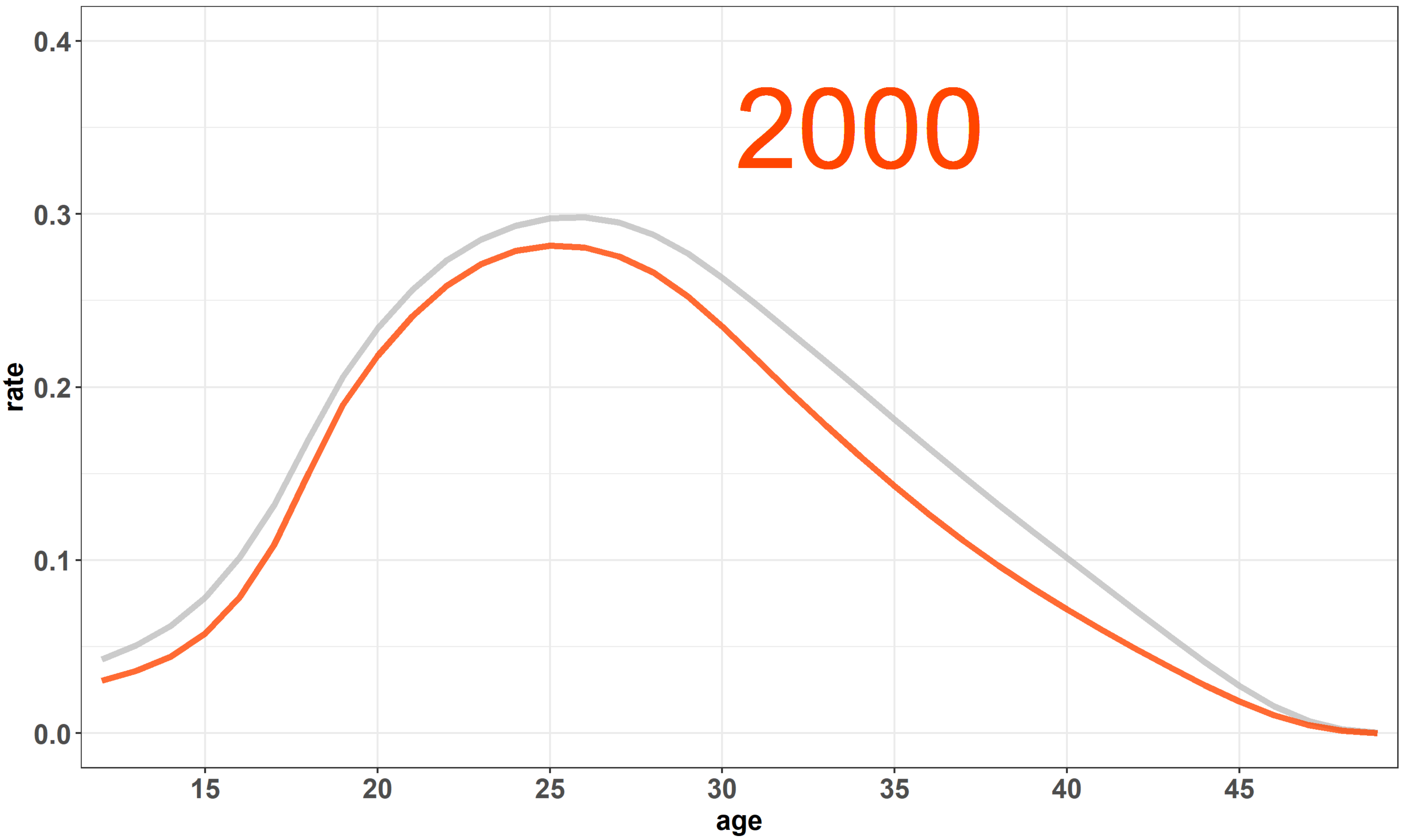

1985
0.4
0.3
0.2
0.1
0.0
rate
15
20
25
30
35
40
45
age



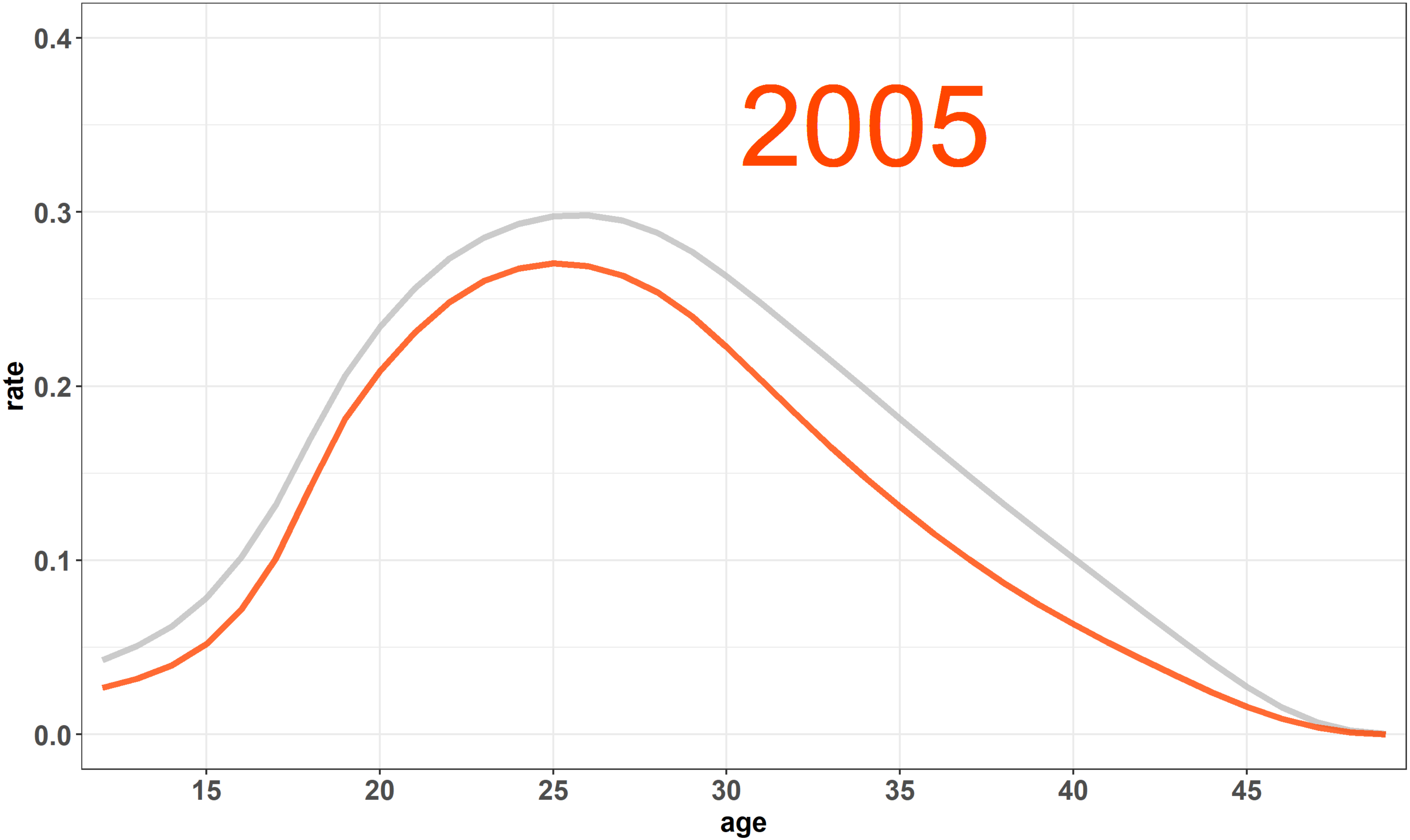
1990
0.4
0.3
0.2
0.1
0.0
rate
15
20
25
30
35
40
45
age

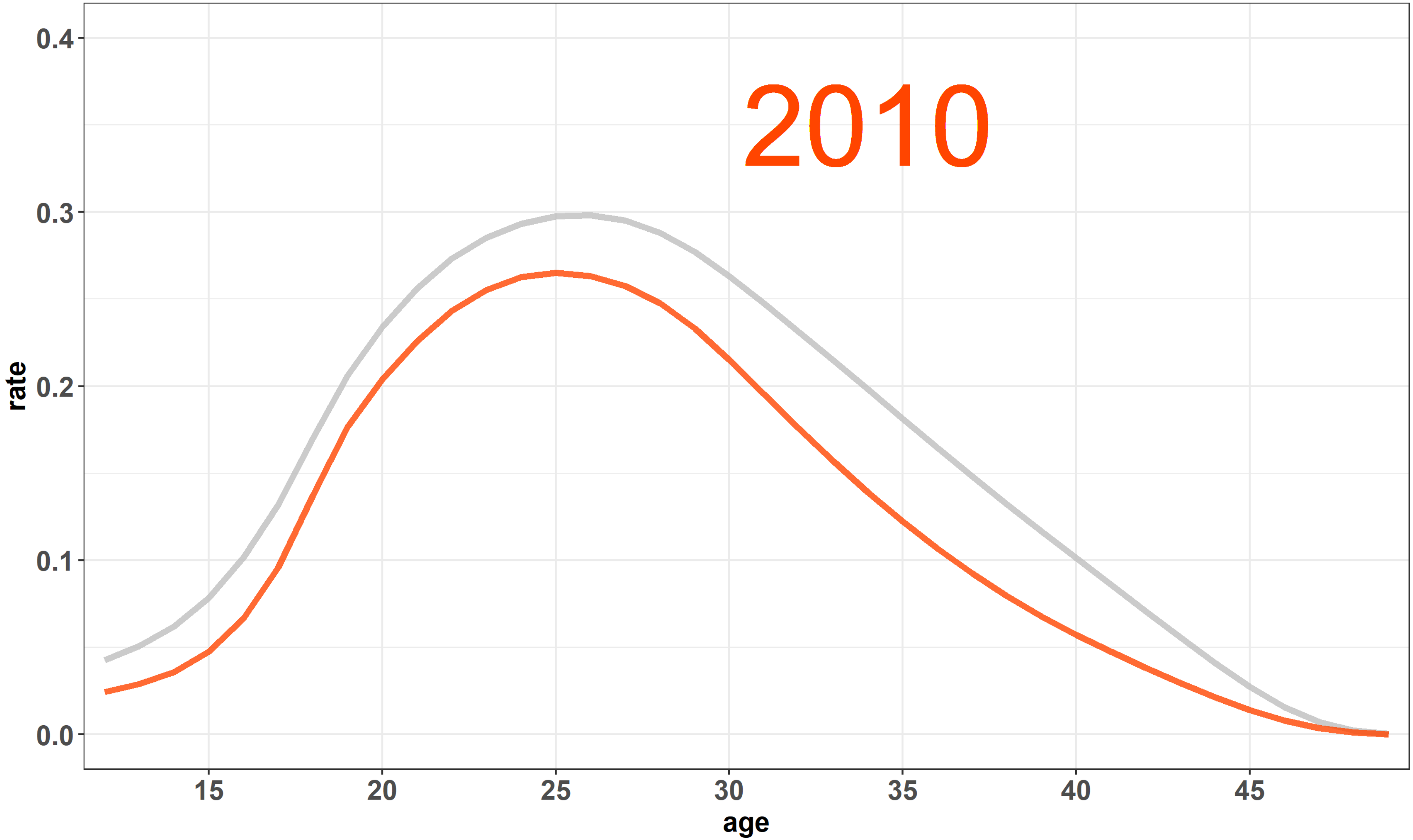


1995
0.4
0.3
0.2
0.1
0.0
rate
15
20
25
30
35
40
45
age

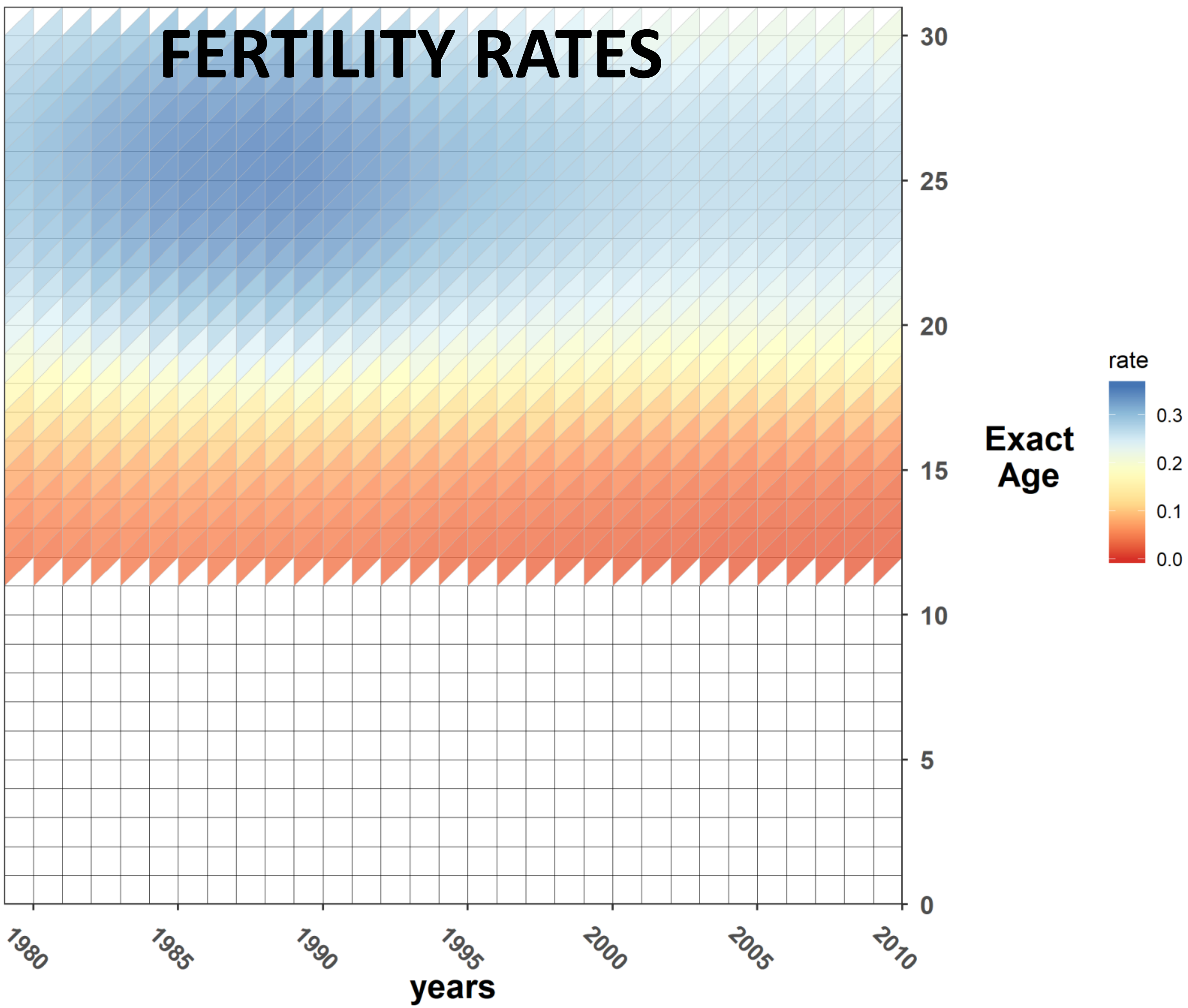


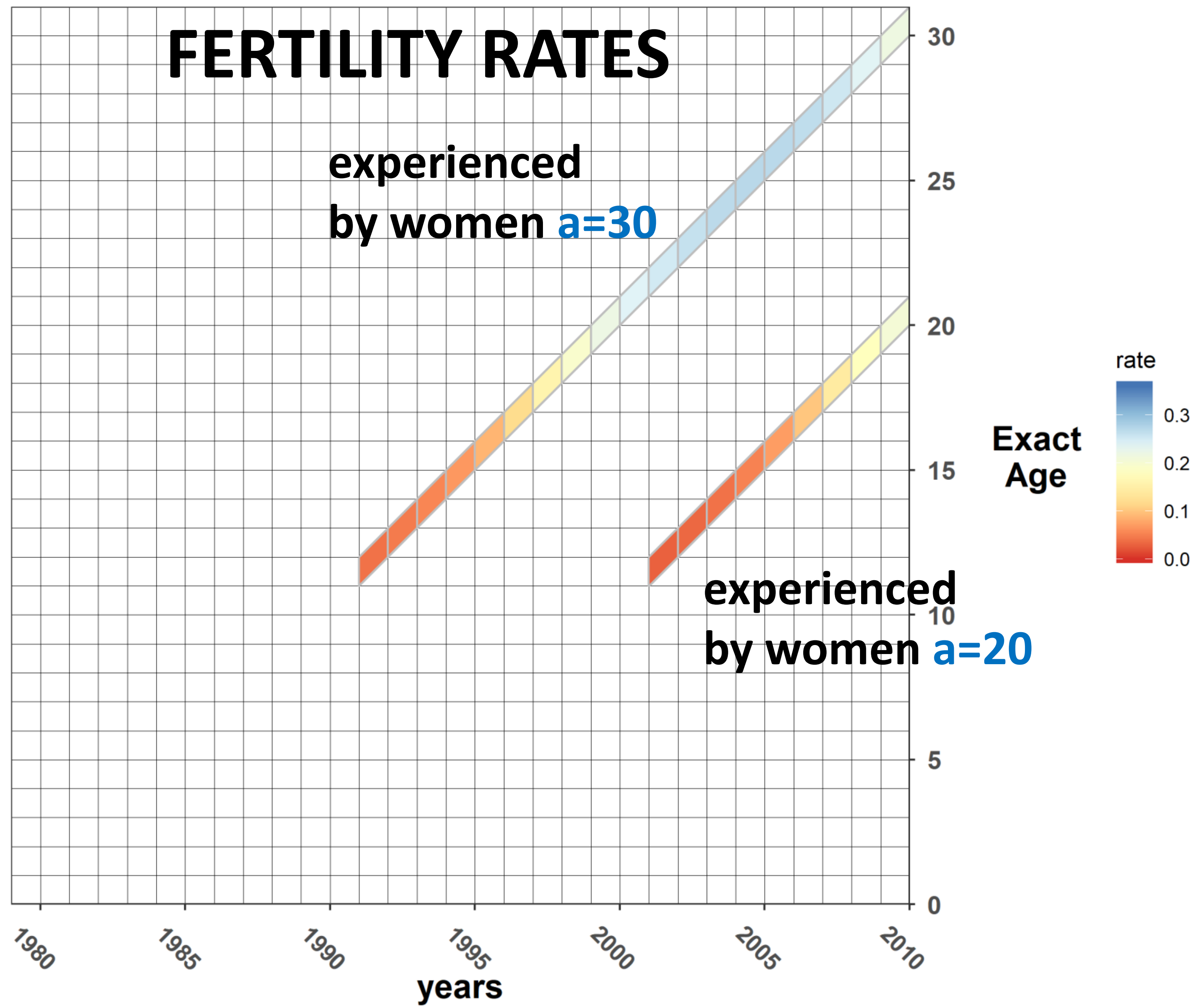
2000
0.4
0.3
0.2
0.1
0.0
rate
15
20
25
30
35
40
45
age


2005
0.4
0.3
0.2
0.1
0.0
rate
15
20
25
30
35
40
45
age



2010
rate
0.4
0.3
0.2
0.1
0.0
age
15
20
25
30
35
40
45


FERTILITY RATES
Exact Age
years
rate
0.3
0.2
0.1
0.0
30
25
20
15
10
5
0
1980
1985
1990
1995
2000
2005
2010


FERTILITY RATES
experienced
by women a=30
experienced
by women a=20
rate
0.3
0.2
0.1
0.0
Exact
Age
30
25
20
15
10
5
0
1980
1985
1990
1995
2000
2005
2010
years

Mortality Model

Unique survival prob p_{xt} for each (age, period)

PARAMETERS

1. Level for period t
   - α_t is the mort level in Clark (2019) model
   - $\alpha_t \approx \text{logit}(q5)_t$ for period t life table
   - $\alpha_t \sim 2^{nd}$ -order random walk, sd=σ_α

Mortality Model (Clark 2019)

Demography (2019) 56:1131–1159
https://doi.org/10.1007/s13524-019-00785-3

Check for updates

A General Age-Specific Mortality Model With an Example Indexed by Child Mortality or Both Child and Adult Mortality

Samuel J. Clark[1,2]

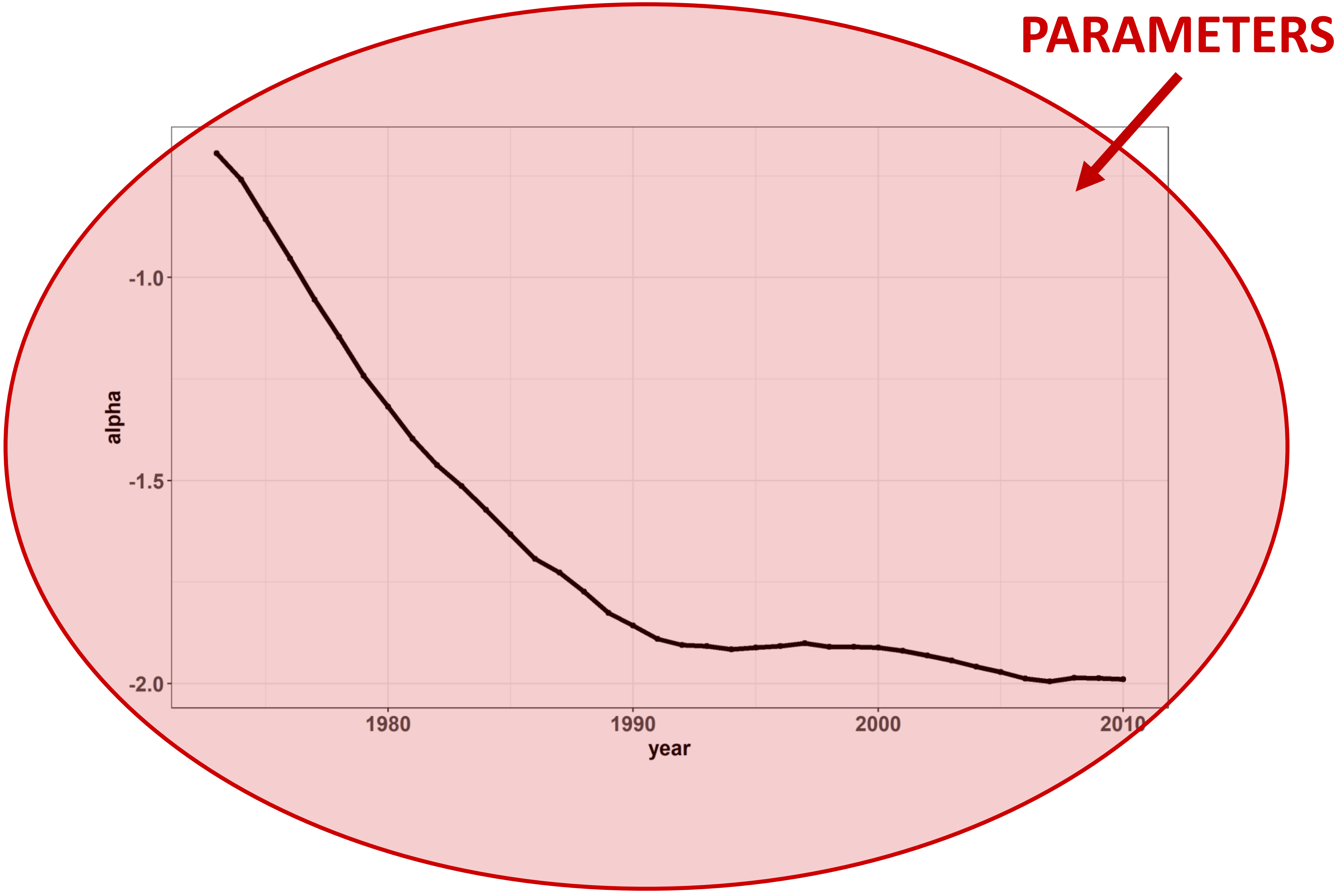
Published online: 28 May 2019

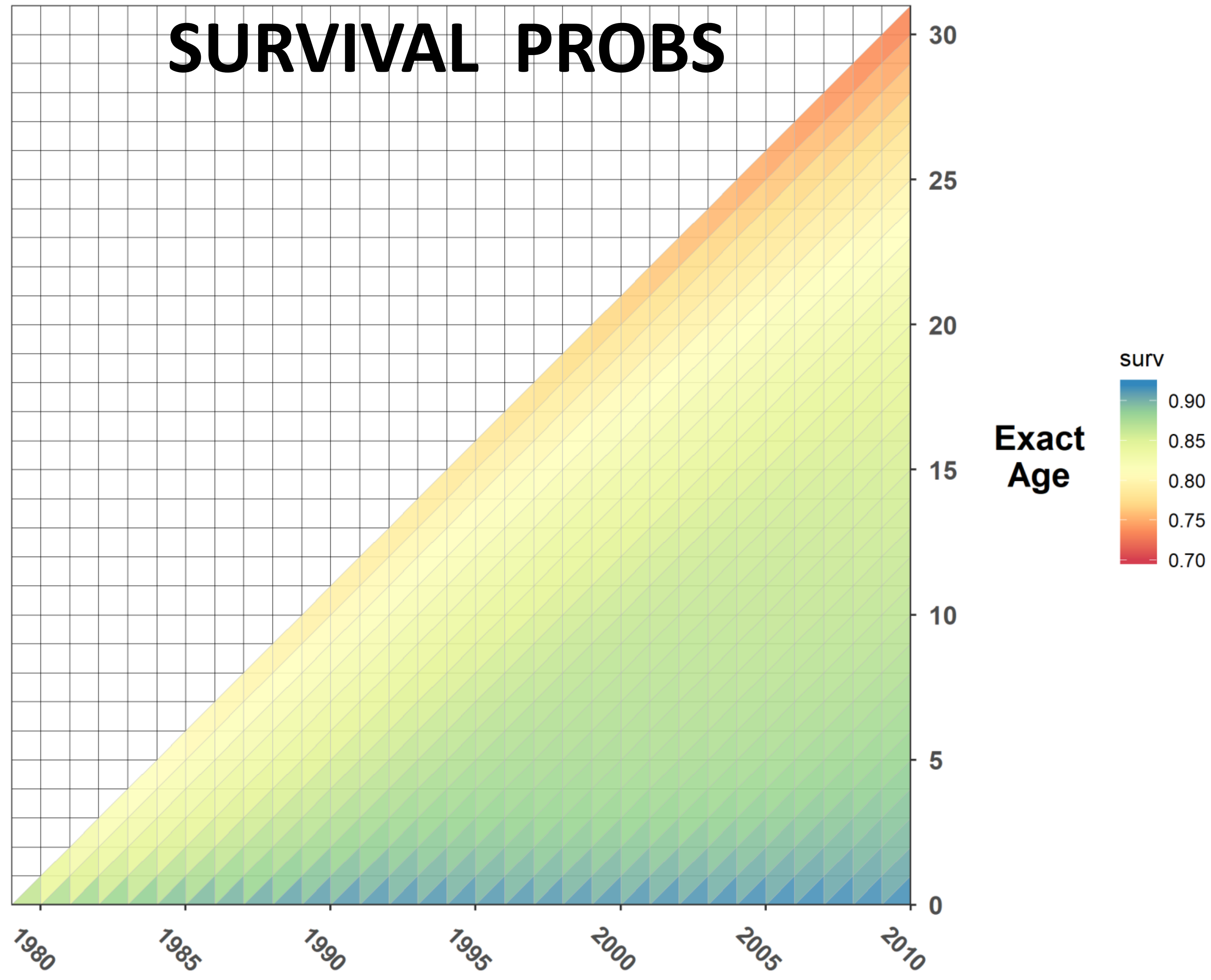
$$\alpha_t \rightarrow \begin{bmatrix} logit({}_1q_0) \\ logit({}_1q_1) \\ \vdots \\ logit({}_1q_{109}) \end{bmatrix}_t \rightarrow \begin{bmatrix} p_0 \\ p_1 \\ \vdots \\ p_{109} \end{bmatrix}_t \rightarrow \left\{ \begin{matrix} cohort \\ survival \\ probs \end{matrix} \right\}$$

Clark Model Constants

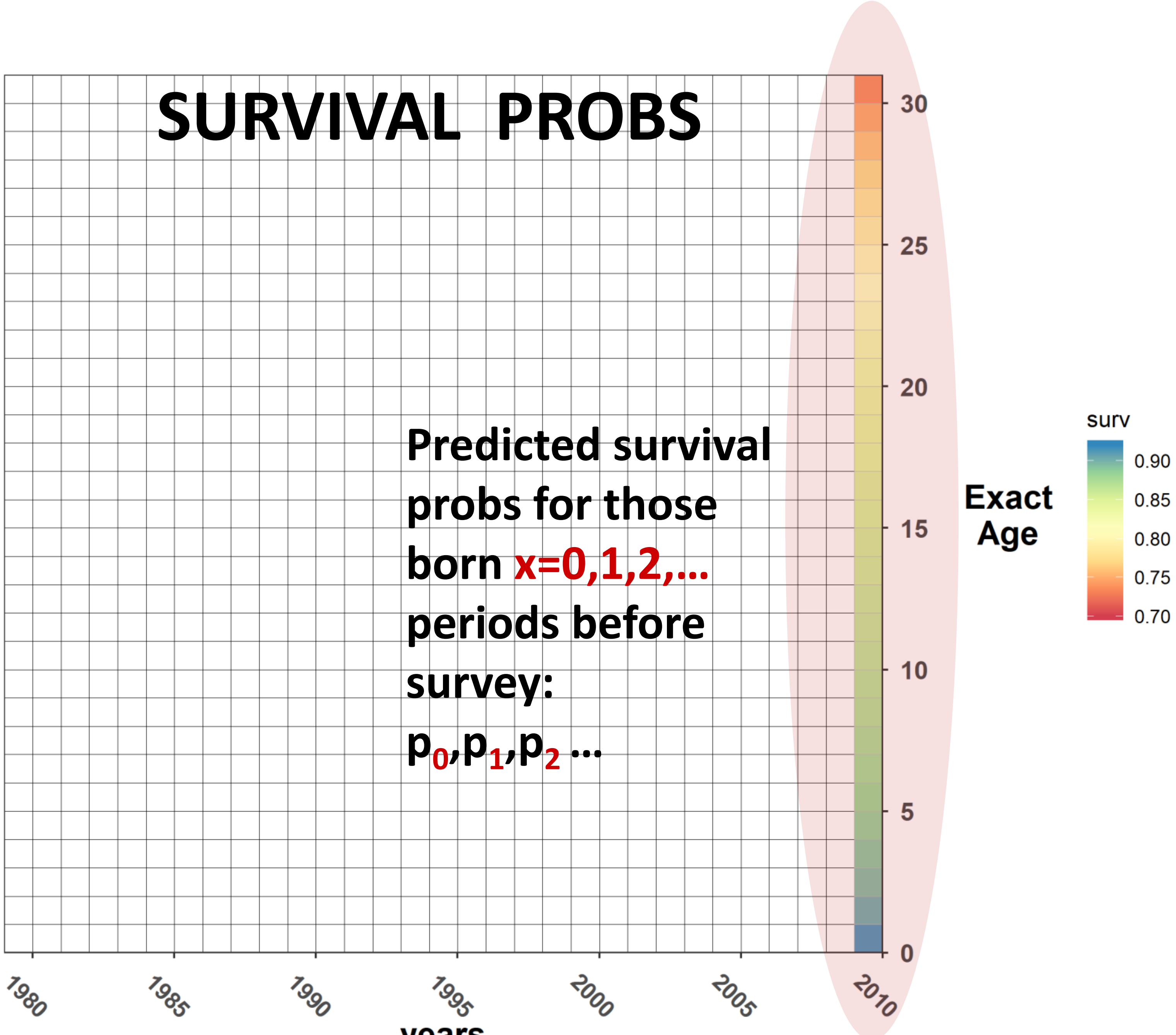
Life Table Relationships

MORTALITY PARAMETERS: EXAMPLE




SURVIVAL PROBS
30
25
20
15
10
5
0
Exact Age
surv
0.90
0.85
0.80
0.75
0.70
1980
1985
1990
1995
2000
2005
2010
years

SURVIVAL PROBS

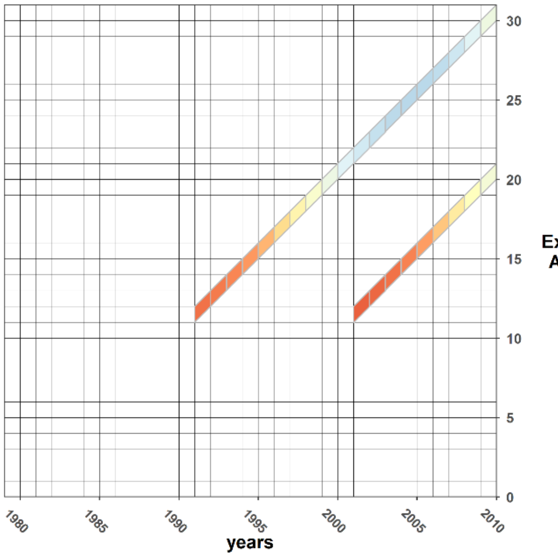


Expected parities and child survival by woman's age

A woman who is $a=28$ yrs old in 2010

was 0 yrs old in 1982

was m yrs old in $1982+m$

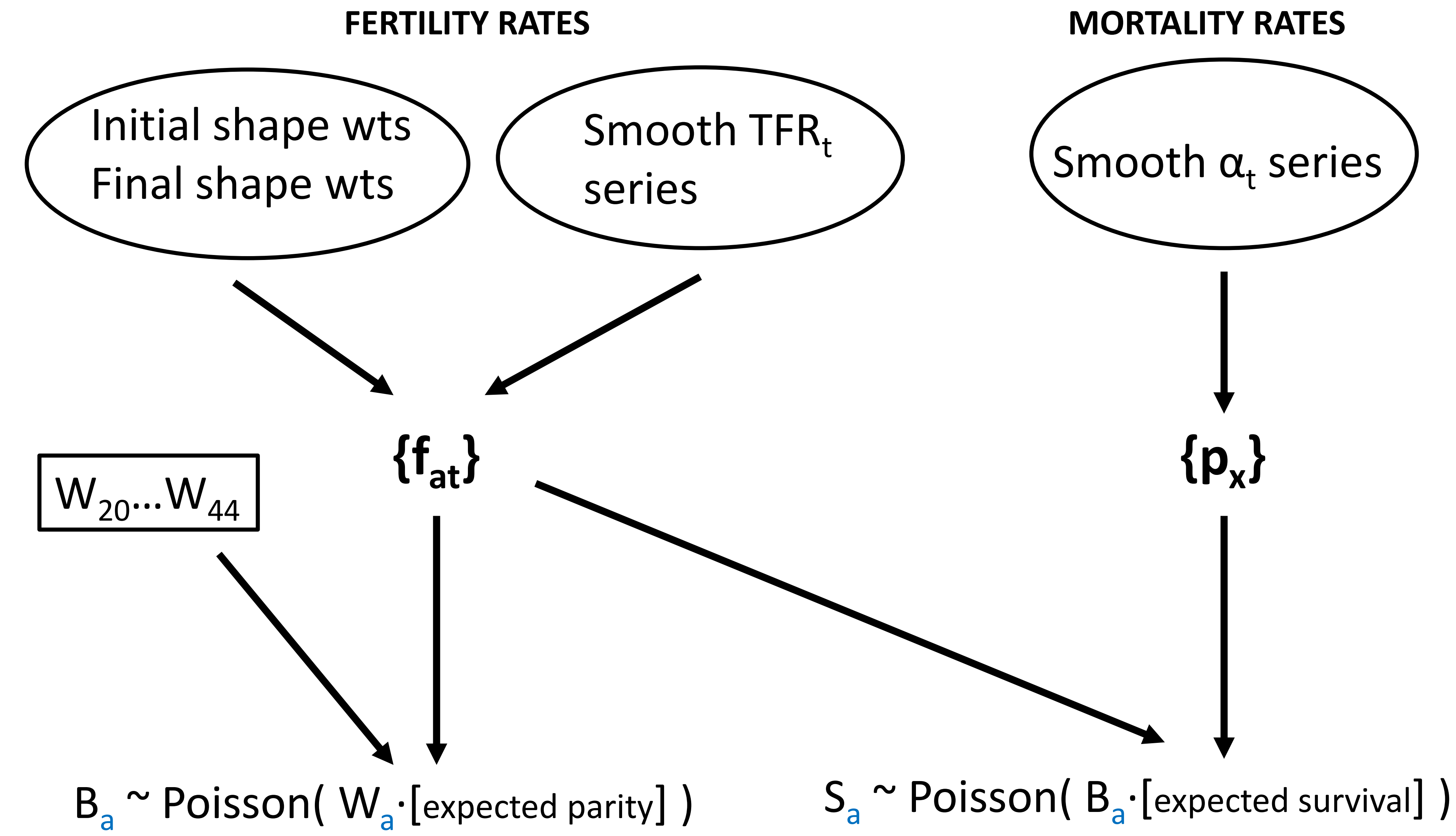


Her expected parity (children ever born) is

$$F_{28} = f_{12,1994} + f_{13,1995} + \ldots + f_{28,2010}$$

Her expected fraction of children surviving is

$$\pi_{28} = (f_{12,1994} / F_{28})\, p_{16}$$
$$+ (f_{13,1995} / F_{28})\, p_{15}$$
$$\ldots$$
$$+ (f_{28,2010} / F_{28})\, p_{0}$$



→ more plausible {fertility, mortality} trends

(1) look like historical patterns [PRIORS]

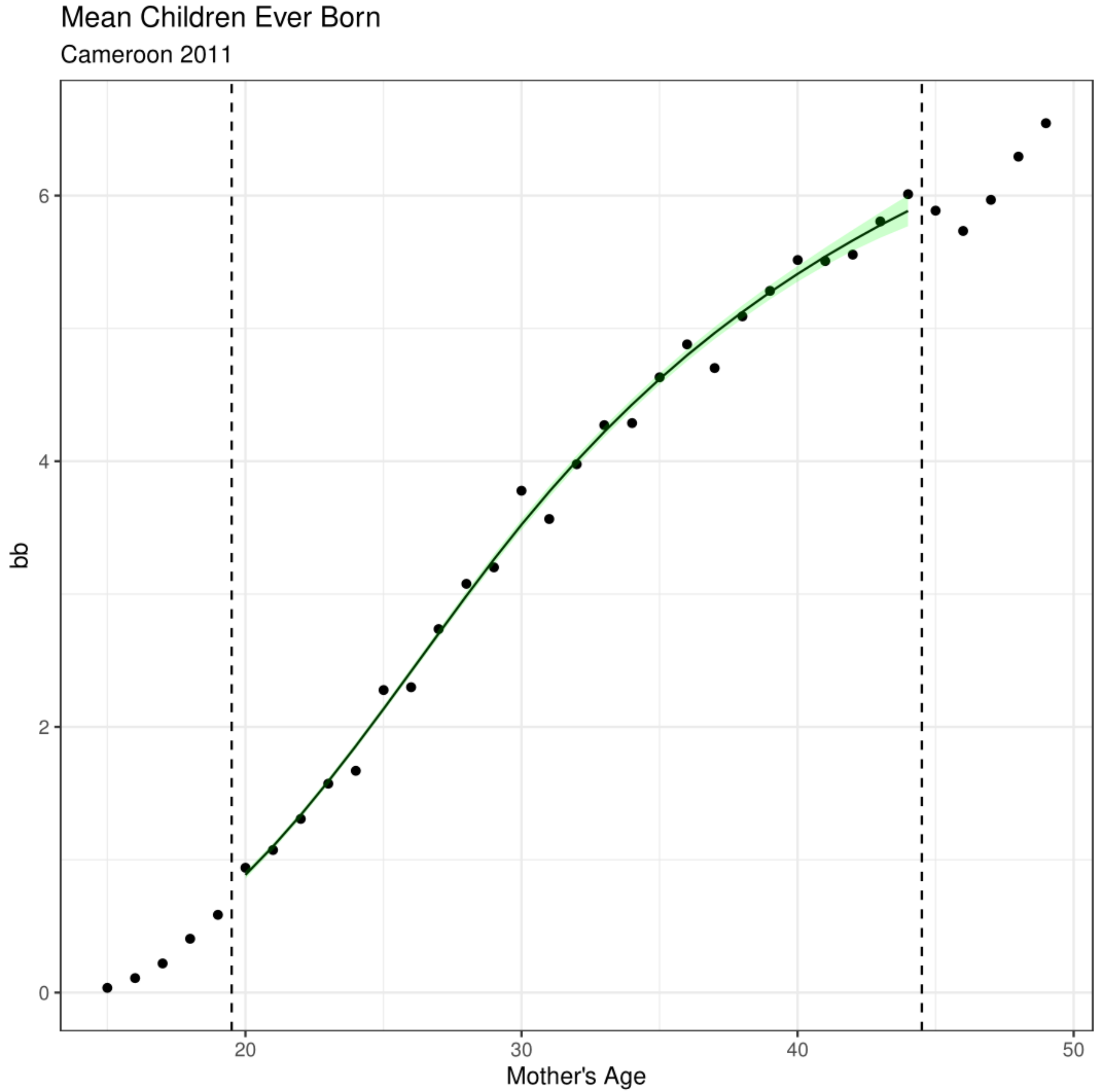
(2) have expected parities & surv that match obs. {B,S} [LIKELIHOOD]

Example Results

Example: Cameroon 2011 DHS

- 15,428 women 15-49
  - Actually have full birth histories (incl. timing of births and deaths)

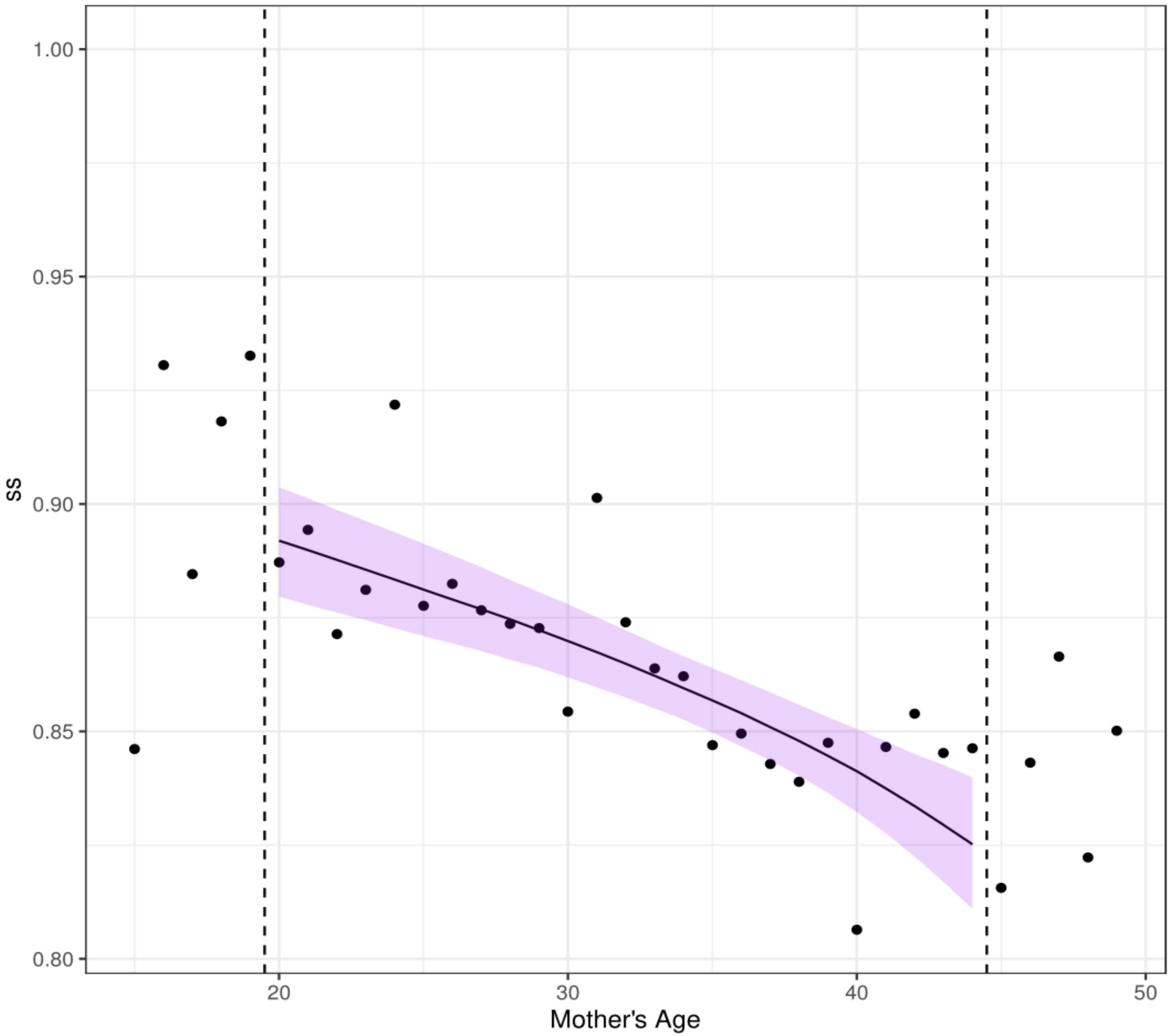
- Summary Birth History Form
  - 42,070 children ever born
  - 5,976 children had died

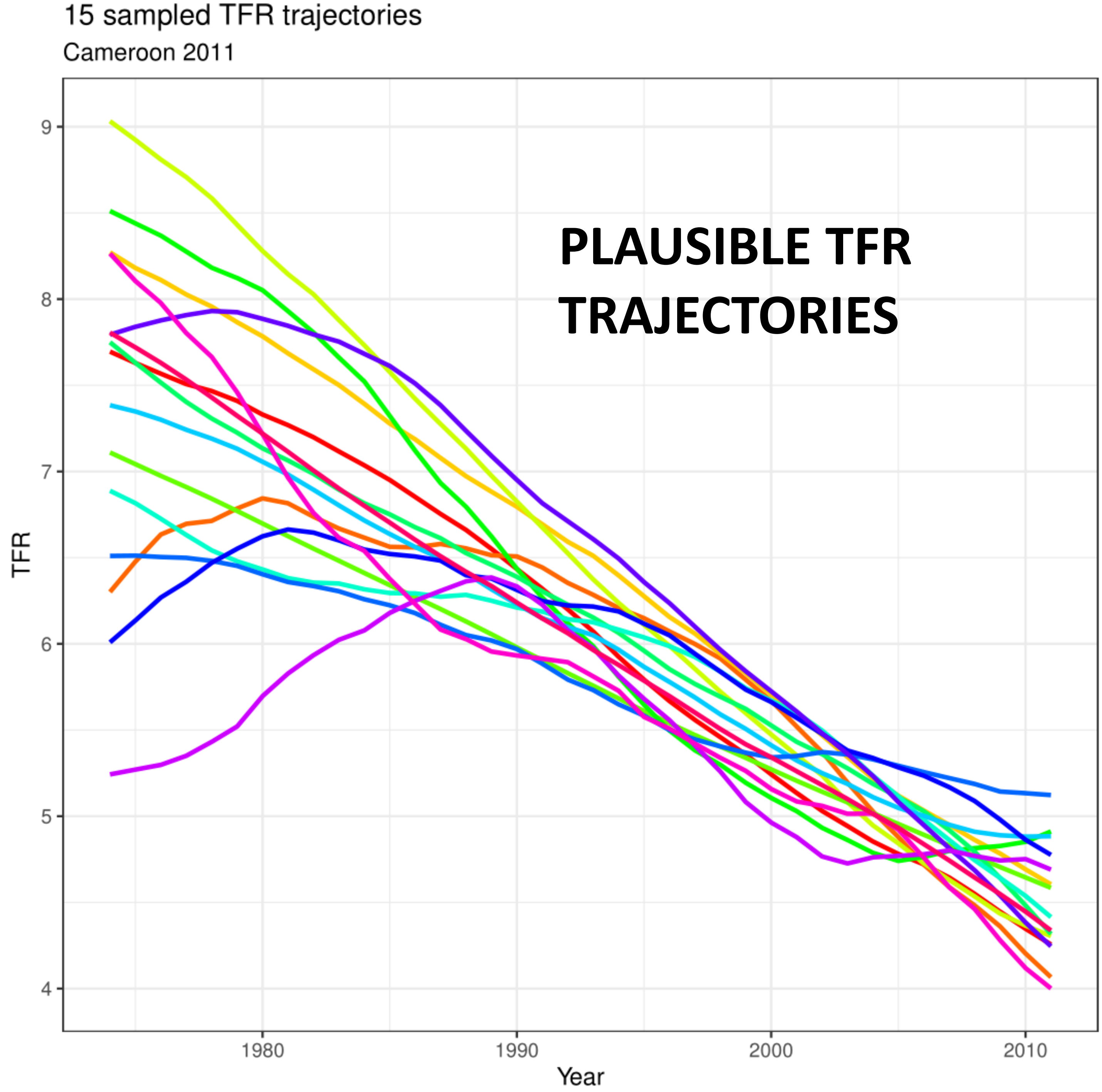


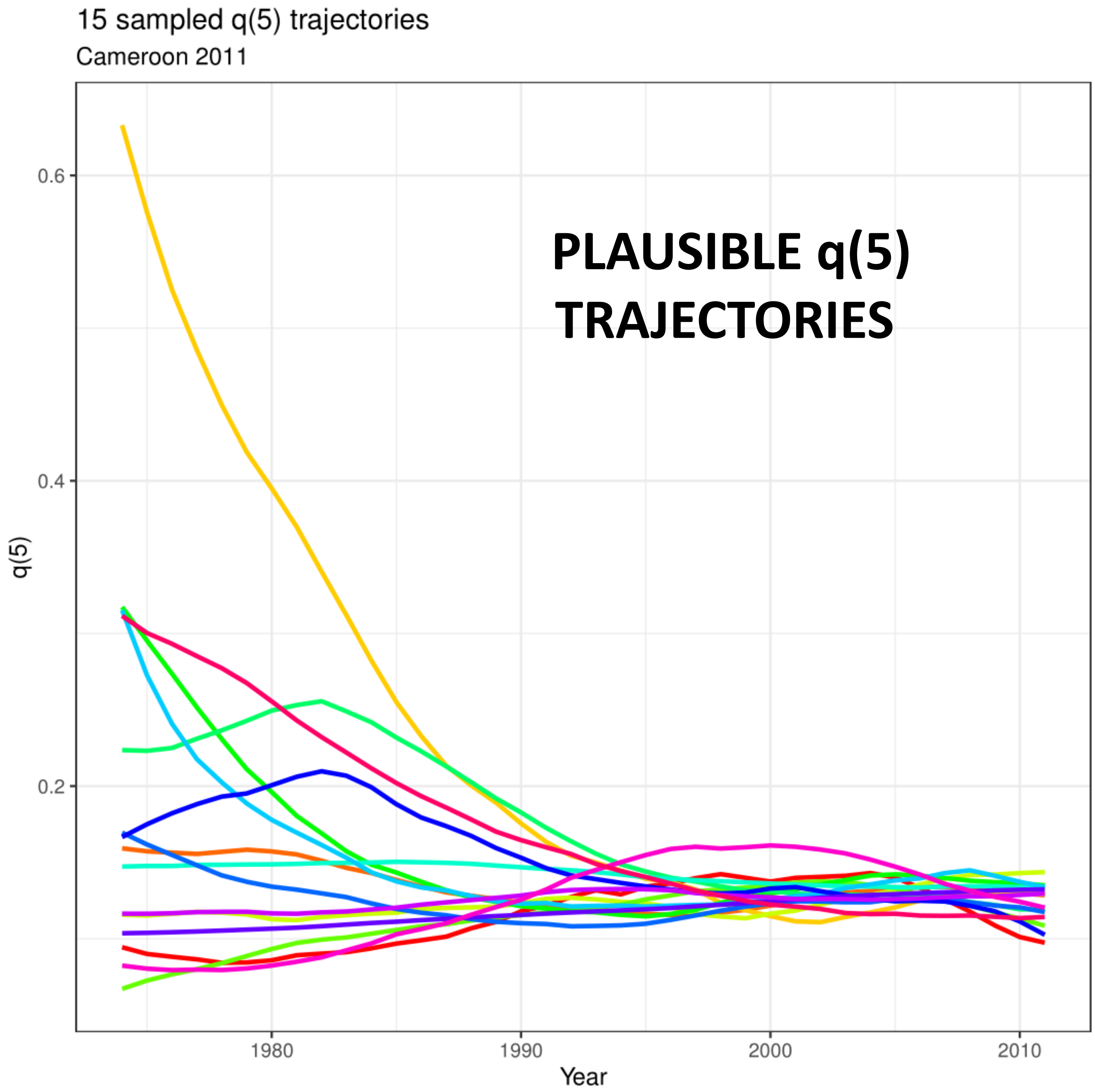
Mean Children Ever Born
Cameroon 2011
bb
0
2
4
6
20
30
40
50
Mother's Age

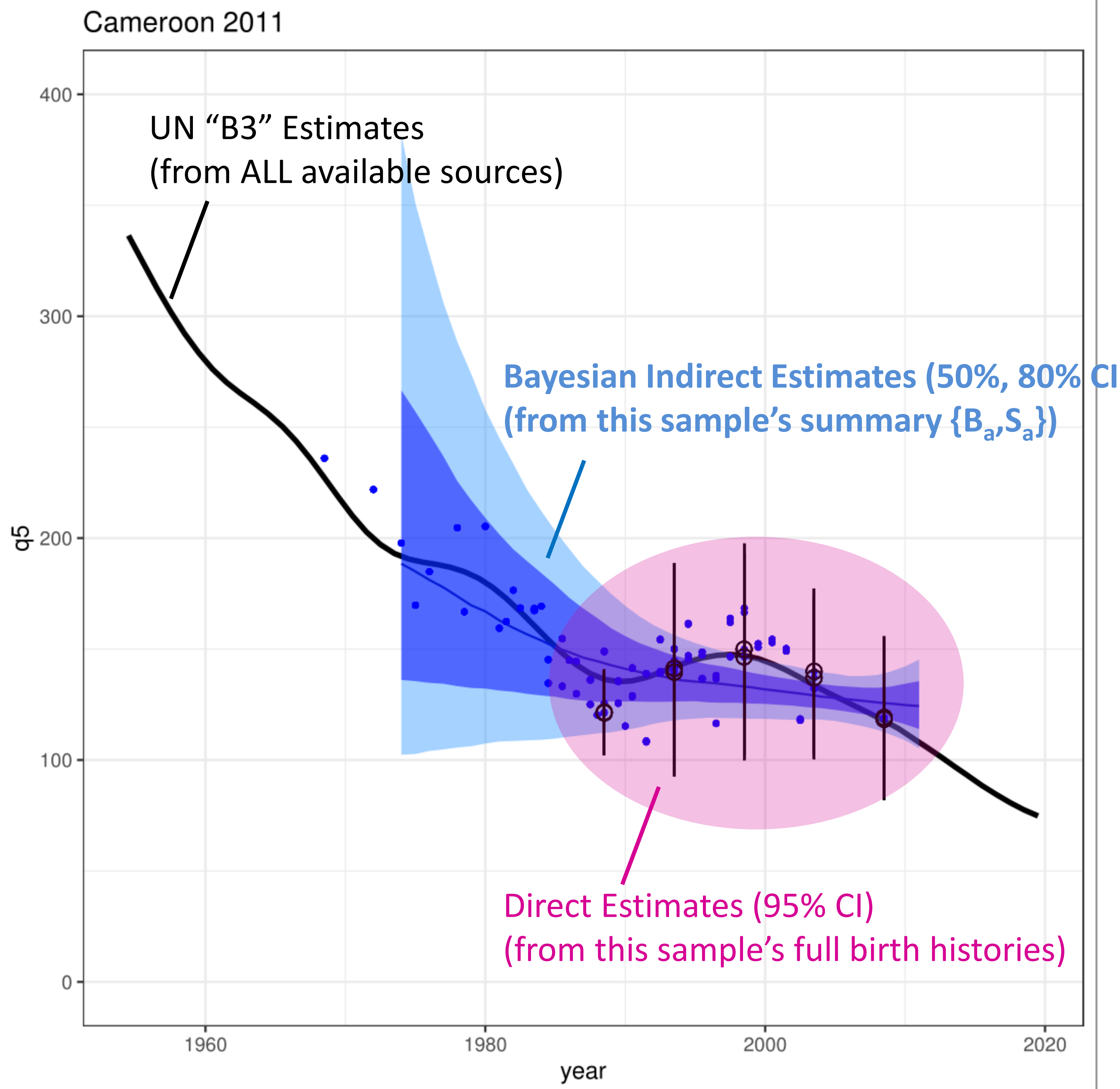
Proportion of Children Alive

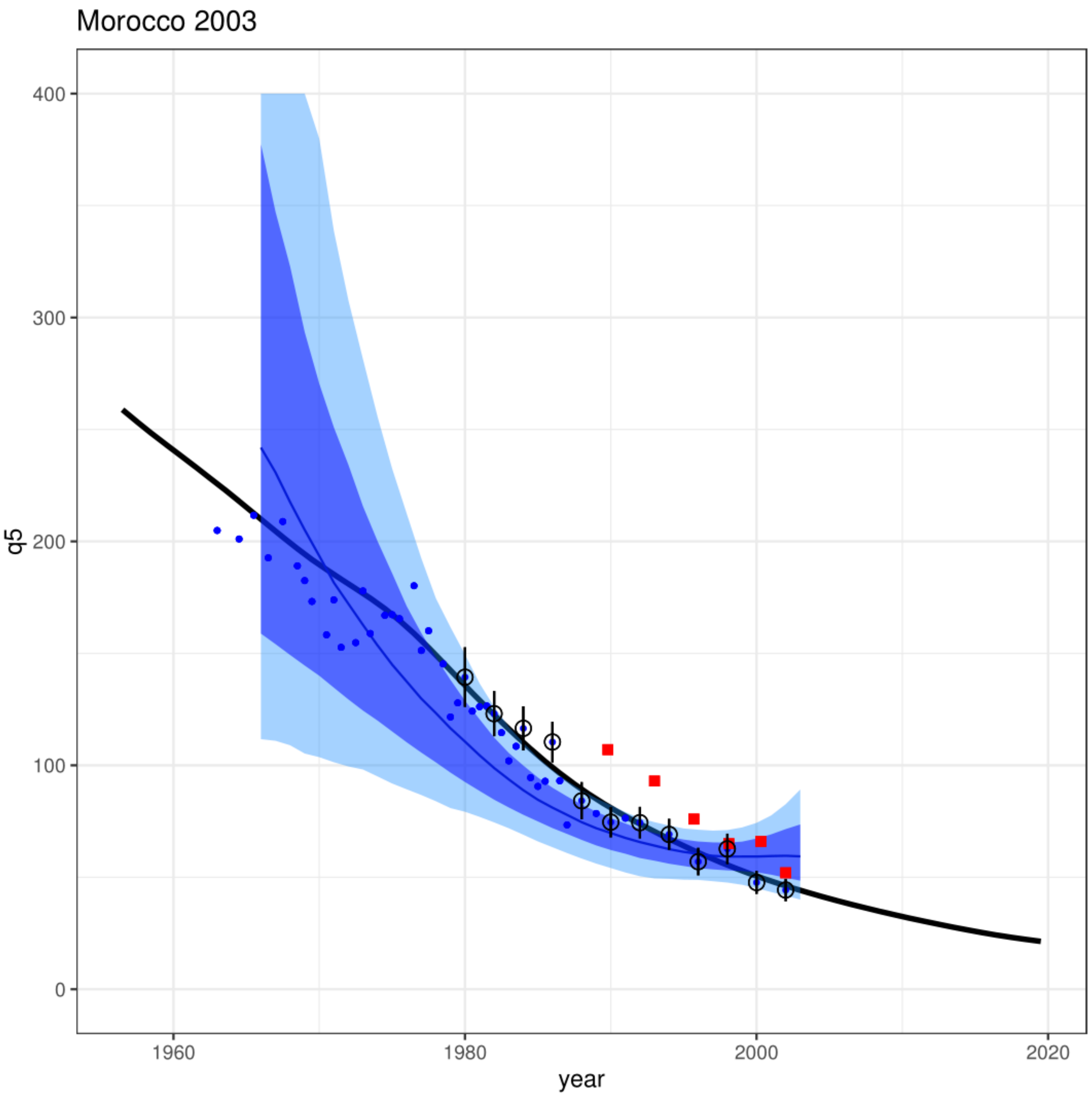
Cameroon 2011



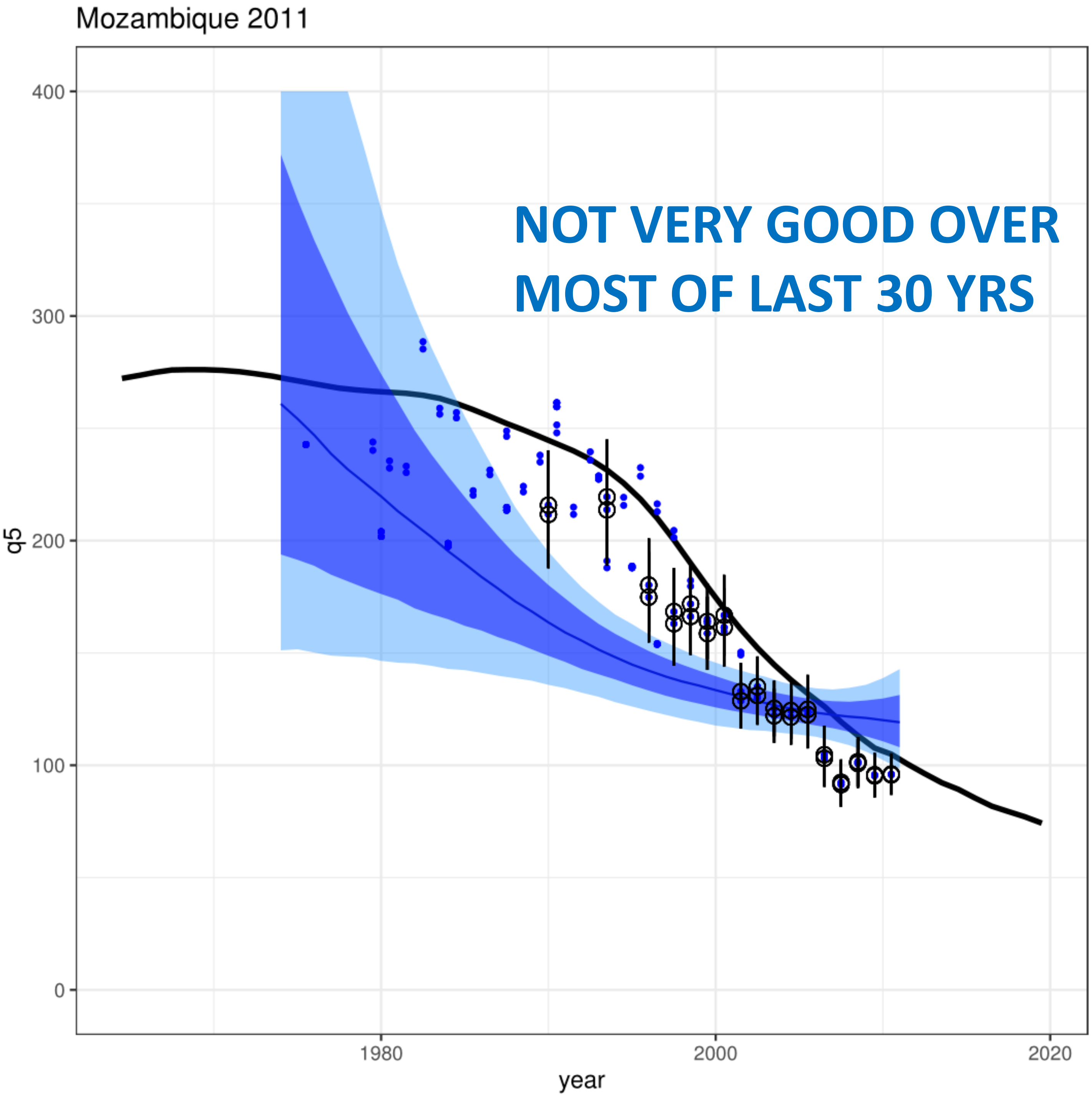

15 sampled TFR trajectories
Cameroon 2011
PLAUSIBLE TFR
TRAJECTORIES
TFR
9
8
7
6
5
4
1980
1990
2000
2010
Year


15 sampled q(5) trajectories
Cameroon 2011
PLAUSIBLE q(5)
TRAJECTORIES
q(5)
0.6
0.4
0.2
1980
1990
2000
2010
Year

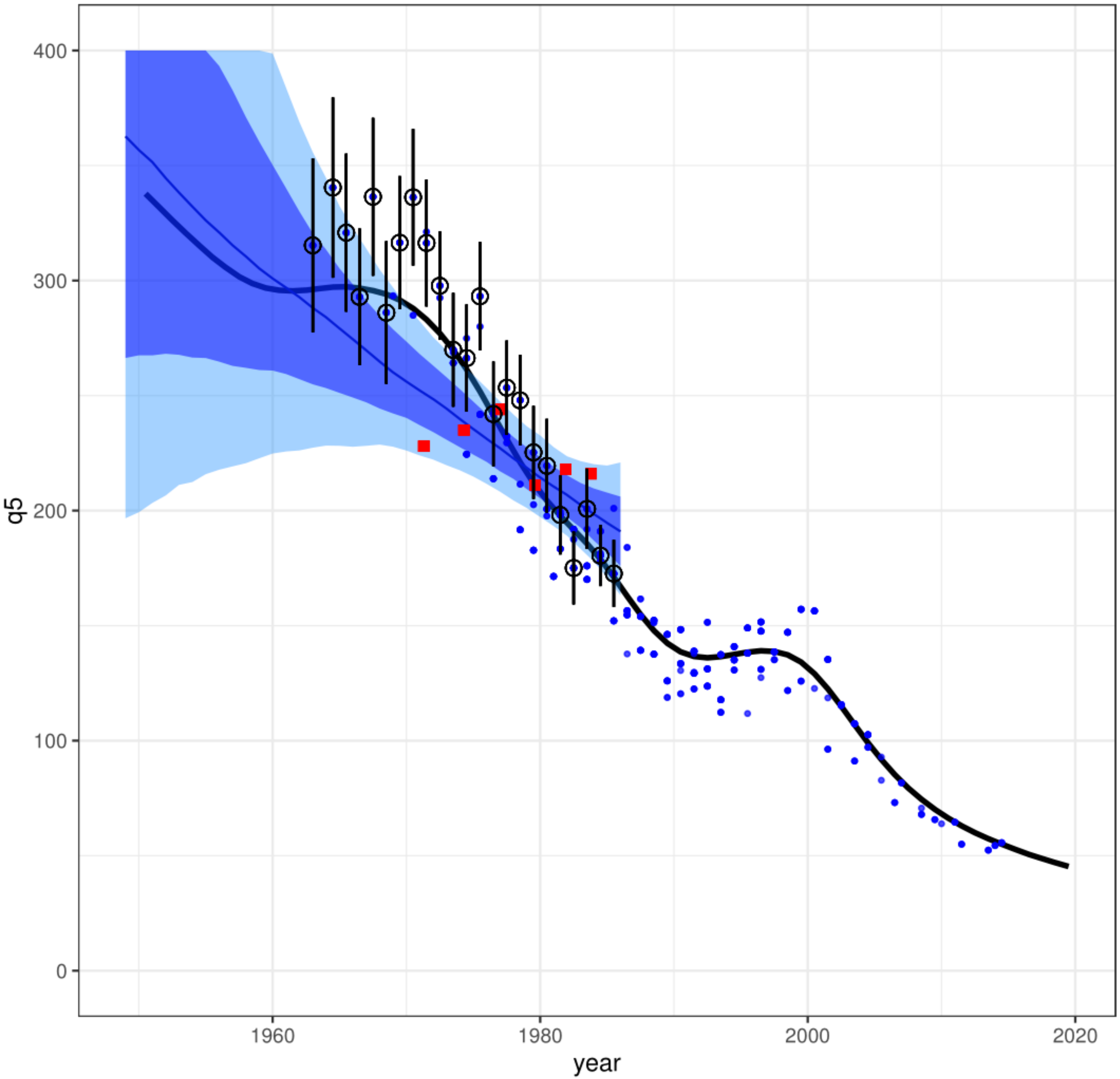

Cameroon 2011
UN "B3" Estimates
(from ALL available sources)
Bayesian Indirect Estimates (50%, 80% CI)
(from this sample's summary {Ba,Sa})
Direct Estimates (95% CI)
(from this sample's full birth histories)
q5
400
300
200
100
0
1960
1980
2000
2020
year



Morocco 2003
q5
400
300
200
100
0
1960
1980
2000
2020
year


Mozambique 2011
NOT VERY GOOD OVER
MOST OF LAST 30 YRS
q5
400
300
200
100
0
1980
2000
2020
year

Senegal 1986



Summary

- Bayesian approach to indirect estimation includes uncertainty about
  - age patterns in fertility rates
  - age patterns in child mortality rates
  - time trends in fertility and mortality
  - sampling noise in (B,S) data

- The approach produces probabilistic estimates of
  - under-five mortality
  - TFR
  - time trends in rates

- Still in progress: In most (but not all) cases the model matches alternative under-five mortality estimates well

THANKS!

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