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Source: *Journal of the American Statistical Association*, Vol. 70, No. 350 (Jun., 1975), pp. 295-304

Published by: Taylor & Francis, Ltd. on behalf of the American Statistical Association

Stable URL: https://www.jstor.org/stable/2285812

Natural Fertility, Population Cycles and the Spectral Analysis of Births and Marriages

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Birth and marriage series are viewed as simultaneous mixed autoregressive and moving average processes with known coefficients, subject to random rates of nuptiality and marital fertility. The theoretical spectra and cross-spectrum are calculated under various hypotheses and compared to empirical estimates for historical birth and marriage series. We reject the "natural fertility" hypothesis and conclude that nuptiality and marital fertility were equally variable and very highly correlated. We also use the model to explain how the age structure of reproduction and nuptiality and duration structure of marital fertility act as filters which transform white noise variation into 30-year cycles in series of births and marriages.

1. INTRODUCTION

A vast amount of demographic data is available in the form of long time series, covering periods ranging from the Middle Ages up to the present. This collection has recently been enlarged by the addition of series of baptisms, burials and marriages taken from parish registers around the world, but particularly from preindustrial Europe. However, the methods used to analyze these data have scarcely advanced since G.U. Yule's careful studies [37] at the turn of the century. Inference is often quite casual; and when techniques such as multiple correlation are used, it is with little attention to the special characteristics of the data. Proper analysis requires careful consideration of the demographic structures generating the series, together with appropriate statistical techniques.

In the last few years, there has been a growing interest in the possibility of using spectral methods to analyze these data (see [7, 11, 24, 25, 29]). In this article, we show how explicit stochastic models of the demographic generating mechanisms, expressed as joint autoregressive processes, may be combined with theoretical and empirical spectral techniques to analyze the behavior of birth and marriage series and test substantive hypotheses. In particular, we will investigate the natural fertility hypothesis and develop the theory of demographic cycles, topics which we now discuss briefly.

It is often asserted that, in preindustrial Europe, marital fertility was relatively constant at "natural" levels and not subject to voluntary control. Year-to-year changes in number of births were due to changes in the age and frequency of marriage (nuptiality) according to this argument (see, e.g., [35, p. 488]). Others have disputed these assertions [2, 5, 26]. Using time series of births and marriages, we will test four hypotheses concerning this controversy. Hypothesis A, the strong natural fertility hypothesis, asserts that marital fertility is constant and all variation in the birth and marriage series originates in fluctuating nuptiality. Hypothesis B asserts the opposite, namely, that nuptiality is constant and all variation is due to marital fertility. Hypothesis C, the weak natural fertility hypothesis, asserts that nuptiality and marital fertility both vary but do so independently, presumably under the influence of different disturbing forces. While nuptiality is controlled voluntarily, marital fertility might, for example, be influenced by involuntary fecundity impairment. Finally, according to Hypothesis D, variations in nuptiality and marital fertility are highly correlated, presumably in response to the same social and economic changes. This suggests voluntary control of fertility within marriage, although other interpretations are possible.

Using a stochastic model of population renewal, we derive the spectral implications of each of these hypotheses and compare them with estimated cross-spectra of actual birth and marriage series.

The spectral analysis of this model also sheds light on the explanation of 30-year cycles in preindustrial series of births and marriages. Two explanations of these cycles have been advanced in the demographic literature: one based on the succession of generations, another on the autocorrelation imparted to variations in births by the persisting effects of nuptiality. We construct a more complete analysis from these and other components.

2. THE BASIC MODEL

Consider series of births (B_t) and marriages (M_t) which, although subject to considerable stochastic variation, do not change much in mean value over long periods of time—on the order of one or two centuries (for the classic example of Crulai, see [10, pp. 60, 61]). If

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© Journal of the American Statistical Association
June 1975, Volume 70, Number 350
Applications Section

all births are legitimate,[1] then the two series will be related as follows:

$$B_t = \sum F_{x,t} M_{t-x} \tag{2.1}$$

$$M_t = \sum W_{a,t} B_{t-a} \tag{2.2}$$

where $F_{x,t}$ is the fertility rate for marriages of x years duration, net of marital dissolution by death or divorce,[2] and $W_{a,t}$ is the female net nuptiality function, describing the distribution by age of female marriages, of all orders and net of mortality, to a birth cohort inclusive of males. Equations (2.1) and (2.2) may be solved for the renewal equation in births alone. After rearranging terms we find:

$$B_t = \sum B_{t-a} (\sum F_{x,t} W_{a-x,t-x}) = \sum B_{t-a} \phi_{a,t} \ . \tag{2.3}$$

Evidently the term in parentheses must approximate the net maternity function, $\phi_{a,t}$.[3]

Let B, M, F_x, W_a and ϕ_a represent long-run average values, and let F and W be the sums of F_x over x and W_a over a, respectively.[4] Then F will be the average number of births per marriage, and W will be the average number of female marriages eventuating per birth.[5] From (2.1) and (2.2) and the assumption of stationary means, we note that $B = FM$ and $M = WB$. Together these imply that $FW = 1$, which reflects long-run demographic stationarity, and, by (2.3), can be seen as equivalent to a net reproduction rate of unity.[6] In a given year, net marital fertility and net nuptiality will differ from F and W; these temporal variations may be measured by F_t (the sum over x of $F_{x,t}$) and W_t (the sum over a of $W_{a,t}$). We will interpret these as variations in marital fertility and nuptiality, although they of course also reflect changes in mortality and, in the case of F, age at marriage, a subject discussed in Appendix A.

From (2.3) we may regard B_t as a function of B_{t-a}, $W_{a-x,t-x}$ and $F_{x,t}$ for each a and x, and the given t. To derive the first-order Taylor approximation for B_t in (2.3), we first take derivatives with respect to B_{t-a}, $F_{x,t}$ and $W_{a-x,t-x}$ for each a and x. These derivatives, evaluated at the mean, are:

$$\partial B_t / \partial B_{t-a} = \sum_x F_x W_{a-x} \doteq \phi_a \ , \tag{2.4}$$

$$\partial B_t / \partial F_{x,t} = \sum_a W_a B = BW = M \ , \tag{2.5}$$

$$\partial B_t / \partial W_{a-x,t-x} = BF_x \ . \tag{2.6}$$

The Taylor approximation is then:

$$B_t \doteq B + \sum_a \phi_a (B_{t-a} - B) + M \sum_x (F_{x,t} - F_x) + B \sum_x F_x \sum_a (W_{a-x,t-x} - W_{a-x}) \ . \tag{2.7}$$

Note that

$$\textstyle\sum_x (F_{x,t} - F_x) = F_t - F$$

and that

$$\textstyle\sum_a (W_{a-x,t-x} - W_{a-x}) = W_{t-x} - W \ .$$

Equation (2.7) can be further simplified by expressing it in terms of proportional variations about the mean. Let $b_t = (B_t - B)/B$; $f_x = F_x/F$; $\theta_t = (W_t - W)/W$; $\epsilon_t = (F_t - F)/F$. Then we have:[7]

$$b_t = \sum c_a b_{t-a} + \sum f_x \theta_{t-x} + \epsilon_t \ . \tag{2.8}$$

This decomposes proportional variations in births into three approximately additive sources: past variations in births (i.e., current population age structure), past variations in nuptiality (i.e., current duration structure of marriages) and current variation in marital fertility.

This stochastic model of the renewal process differs from others, e.g., [20, 32], by distinguishing between variation originating in nuptiality, (θ_t), which has persisting effects, and in marital fertility, (ϵ_t), which has a transitory effect. Implications for the cyclic behavior of the birth series will be explored in Section 7.

Note that by symmetry an analogous decomposition holds for proportional variations in marriages. Letting $c_0 = 1$, and $c_j = -\phi_j$, we may express this pair of simultaneous processes:

$$\sum c_j b_{t-j} = \sum f_x \theta_{t-x} + \epsilon_t \tag{2.9}$$

$$\sum c_j m_{t-j} = \sum w_a \epsilon_{t-a} + \theta_t \ , \tag{2.10}$$

where $m_t = (M_t - M)/M$ and $w_a = W_a/W$.[8]

Using (2.9) and (2.10), we can develop the variance and covariance structure of series of births and marriages under the hypotheses discussed in Section 1. According to Hypothesis A, marital fertility is constant ($\sigma_\epsilon^2 \doteq 0$) so that all variation in births and marriages originates in fluctuating nuptiality ($\sigma_\theta^2 > 0$). This is the "strong natural fertility hypothesis." Under Hypothesis B,

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[1] If premarital conceptions lead to marriages, then F_x will be positive for some x less than zero. In Crulai, there was very little premarital conception, and less than 1 percent of births were illegitimate. In 19th century Sweden, 5 or 6 percent were illegitimate, and in some other areas the proportion was much higher.

[2] Such a function has been used occasionally to correct a birth series for the influence of fluctuations in a parallel marriage series (see [2, 5, 6, 37]). It is discussed more generally in [28]. The function evidently will depend on the age distribution of the female marriage cohort to which it refers and on rates of mortality and divorce, as well as on marital fertility. See Appendix A.

[3] Calculations show that this approximation is quite good, although since we have attributed the same stream of births to a bride aged 20 as one aged 50, there is too much weight given to the older ages.

[4] We would like to interpret these as expected values, but this would involve difficult problems. Covariance stationarity, assumed in subsequent sections requires that the net reproduction rate, given by $\Sigma\phi_a$, be less than unity. But in this case the intrinsic rate of increase is negative, and the expected values of births and marriages would be zero. The problem may be avoided in a conceptually natural way by altering the model so that it equilibrates at a positive level, as is done in [24]. This would complicate the presentation without altering the results, except for very long-run variations.

[5] In 18th century western Europe, F was roughly 4 or 5, and W was roughly .20 to .25. W reflects the sex ratio at birth and survival probabilities as well as nuptiality.

[6] The net reproduction rate is defined as the sum over all ages of the net maternity function, ϕ_a.

[7] The effect of ignoring second-order terms depends on the amplitude of the variations and on their distribution by frequency (spectrum), with large amplitudes and low frequencies leading to larger errors. The effect of ignoring higher-order terms in the deterministic renewal equation has been examined by Coale [3], who found it small in most cases.

[8] This representation of the renewal process can be used to forecast jointly series of births and marriages, following the rationale and method developed in [22].

nuptiality is constant ($\sigma_\theta^2 \doteq 0$), and all variation originates in marital fertility ($\sigma_\epsilon^2 > 0$). Hypothesis C asserts that nuptiality and marital fertility vary independently, in response to completely different stimuli ($\sigma_\theta^2 > 0$; $\sigma_\epsilon^2 > 0$; $\sigma_{\theta,\epsilon} \doteq 0$). Hypothesis D is most general, attributing some variation to mutually exclusive factors, while allowing for another component of variation due to factors affecting both nuptiality and marital fertility, perhaps with a lag. This hypothesis may be represented as:

$$\theta_t = \theta_t' + \eta_t \tag{2.11}$$

$$\epsilon_t = \epsilon_t' + \sum k_j \eta_{t-j} \tag{2.12}$$

where θ_t', ϵ_s' and η_r are uncorrelated for all r, s, t. For example, if the reactions of marriages and *conceptions* to a common stimulus are roughly coincident, then fertility will lag the response of marriages by about nine months. In this case, $k_0 = .25$, $k_1 = .75$, and $k_j = 0$ otherwise, with $\sigma_\epsilon^2 > 0$, $\sigma_\theta^2 > 0$, $\sigma_\eta^2 > 0$.

So far we have assumed that the series were approximately stationary in the mean over long periods. However, more generally, the means of F_x and W_a will imply a mean net reproduction rate different from unity, and the series B_t and M_t will then follow exponential trends. With appropriate transformations of the functions and variables, the entire analysis carries over to this more general case.[9]

3. THEORETICAL SPECTRAL RELATIONS

We now develop the implications of the stochastic model relating births and marriages for the behavior of the two series. Spectral analysis is particularly useful for this purpose because it decomposes the variance and covariance of the series by frequency, enabling us to consider short-run and long-run effects separately. (For a time-domain approach, see [2 or 23].) We will describe the method briefly (for details, see [16, 13 or 27]).

The spectrum of a process x_t, denoted $g_x(\lambda)$, gives the distribution of the variance of x_t by frequency λ. In Figures A–F, frequency is expressed in cycles per century. A frequency of 20 cycles per century corresponds to a period, or cycle length, of 100/20 or five years. If x is an uncorrelated random series, then its spectrum will be constant over all frequencies. If there is a tendency for x to move in cycles of frequency λ_0, then $g_x(\lambda)$ will show a peak at λ_0.

The relation between two series is described by the cross-spectrum in terms of the "gain," "phase angle" and "coherence." The gain, $G(\lambda)$, indicates the extent to which fluctuations in one series, say x, are amplified or attenuated as they are passed on to another series, y, at a specific frequency. It is analogous to a frequency-specific regression coefficient of y on x. The phase angle, $\phi(y)$, indicates the extent to which the response of y lags or leads the fluctuations of x at frequency λ. We measure it in fractions of a cycle; thus, a phase angle of .1 at a frequency of 25 cycles per century indicates that y lags x by one-tenth of a cycle or .4 years at this frequency.

The coherence, $C(\lambda)$, is analogous to the correlation between the two series at frequency λ; the coherence squared, $C^2(\lambda)$, indicates the proportion of the variance of y that can be explained by x at frequency λ.

All these functions may be calculated either from a theoretical model or actual time series data. Comparison of the two allows testing.

In this Section and in Appendix B, we derive the spectra of b (birth variations) and m (marriage variations), and the coherence squared, gain squared and phase shift relating them. This is done under each of four hypotheses about the variance and covariance of marital fertility (ϵ) and nuptiality (θ). The variances of ϵ and θ are described by their spectra, $g_\epsilon(\lambda)$ and $g_\theta(\lambda)$. Their covariance is described by the spectrum of their common component, $g_\eta(\lambda)$, and by the effect of the lag structure relating ϵ and η, as described by the coefficients k_j and their transfer function $K(\lambda)$ (see (2.12)).

The remainder of this article will be concerned with four illustrative specifications of $g_\epsilon(\lambda)$, $g_\theta(\lambda)$, $g_\eta(\lambda)$ and $K(\lambda)$, corresponding to the four sets of hypotheses introduced in Sections 1 and 2. The numerical values used with each specification are based in part on empirical analysis, discussed in subsequent sections.

Hypotheses A asserts that marital fertility is constant; this is the strong natural fertility hypothesis. Thus, $g_\epsilon(\lambda) = 0$, $g_\eta(\lambda) = 0$, and $g_\theta(\lambda) = 1$.

Hypothesis B asserts that nuptiality is constant; thus, $g_\theta(\lambda) = 0$, $g_\eta(\lambda) = 0$ and $g_\epsilon(\lambda) = 1$.

Hypothesis C asserts that nuptiality and marital fertility vary independently; this is the weak natural fertility hypothesis. Thus, $g_\epsilon(\lambda) = g_\theta(\lambda) = 1$, and $g_\eta(\lambda) = 0$.

Hypothesis D asserts that marital fertility and nuptiality are correlated with an appropriate lag; thus, $g_\epsilon(\lambda) = g_\theta(\lambda) = 1$, $g_\eta(\lambda) = .9$, $k_0 = .67$ and $k_1 = .33$. This particular specification for Hypothesis D was chosen because it fits the behavior of the Swedish series quite well. The specification of k_0 and k_1 implies that ϵ_t lags θ_t by four months or that fluctuations in marital conceptions lead those in nuptiality by five months. The relative values of $g_\eta(\lambda)$, $g_\epsilon(\lambda)$ and $g_\theta(\lambda)$ imply that, for long-run fluctuations, the correlation of ϵ and θ is .9; for shorter

[9] LeBras [20] has shown that a unique rate of increase may be associated with each vector of stochastic stationary age-specific net maternity rates such as our $\phi_{a,t}$, as has Cohen [4] under more restrictive assumptions. This rate will not in general be the intrinsic rate of natural increase implied by the vector of expected values, ϕ_a, but it will be *close* to that rate for processes of practical interest and will equal it when there is no serial correlation (see [32]). The most unfavorable case occurs when net maternity is cyclic with period equal to the mean age of child-bearing; but then even if the amplitude of cycle is .5 when the net reproduction rate is 1.0, the growth rate will deviate by only .0025 from zero (see [20, p. 551] or [3, p. 186]). Thus the growth rate of the stochastic population approximately equals the intrinsic rate of increase, r, implied by ϕ_a (where r is defined by $1 = \exp(-ra)\phi_a$). It follows that the transformed stochastic maternity function, $\phi_{a,t}^* = \exp(-ra)\phi_{a,t}$, will have a mean intrinsic rate of 1, and therefore the series generated by it will be approximately stationary in the mean over the long run. From this it follows that the process generated by $F_{x,t}^* = \exp(-rx)F_{x,t}$ and $W_{a,t}^* = \exp(-ra)W_{a,t}$ will be approximately stationary in the mean, which can be seen by substitution in (2.3). The birth and marriage series generated will be related to the original series by: $B_t^* = \exp(-rt)B_t$ and $M_t^* = \exp(-rt)M_t$. Therefore the entire analysis developed in this article can be applied to functions and series that have been transformed as indicated above. We will assume that such a transformation has been made, and drop the asterisked (*) notation.

fluctuations, the correlation is weakened by the use of annual discrete time units, as expressed by declining values of $|K(\lambda)|^2$ at higher frequencies.[10]

The table, derived from the analysis of Appendix B, presents the spectral and cross-spectral expressions under each of these hypotheses, using an abbreviated notation.

Spectra and Cross-Spectrum of b and m Under Four Hypotheses Concerning Nuptiality and Marital Fertility[a]

Spectrum	A Marital fertility constant	B Nuptiality constant	C Marital fertility and nuptiality independent	D Marital fertility and nuptiality correlated
$g_b(\lambda)$	$a_\lambda^2 f_\lambda^2$	a_λ^2	$a_\lambda^2(f_\lambda^2 + 1)$	$a_\lambda^2[f_\lambda^2 + 1 + \mathrm{RE}(f_\lambda k_\lambda)]$
$g_m(\lambda)$	a_λ^2	$a_\lambda^2 w_\lambda^2$	$a_\lambda^2(w_\lambda^2 + 1)$	$a_\lambda^2[w_\lambda^2 + 1 + \mathrm{RE}(w_\lambda \bar{k}_\lambda)]$
$g_{bm}(\lambda)$	$a_\lambda^2 f_\lambda$	$a_\lambda^2 w_\lambda$	$a_\lambda^2(\bar{f}_\lambda + w_\lambda)$	$a_\lambda^2(\bar{f}_\lambda + w_\lambda + .5\bar{k}_\lambda + .5k_\lambda w_\lambda \bar{f}_\lambda)$

[a] For specification of Hypotheses A, B, C and D, see text. The table uses the following notation:
$a_\lambda^2 = 1/|C(\lambda)|^2$; $f_\lambda = F(\lambda)$; $f_\lambda^2 = |F(\lambda)|^2$; $w_\lambda = W(\lambda)$; $w_\lambda^2 = |W(\lambda)|^2$; $k_\lambda = K(\lambda)$; bar indicates conjugate; RE( ) indicates real part of argument.

The demographic functions c, F and W are known for many populations, and we have used them to calculate numerical values for the theoretical spectral functions. These, along with empirical cross-spectral estimates, will be discussed later.

4. THE INFLUENCE OF MARRIAGES ON BIRTHS

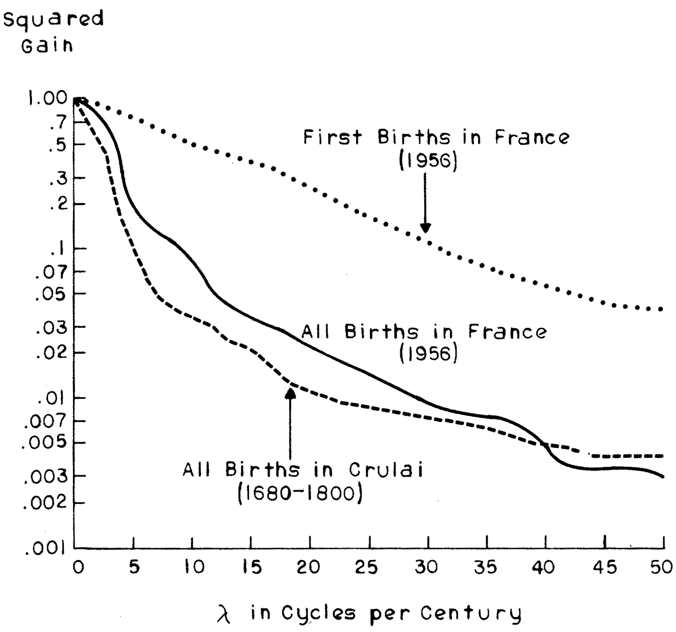
Before applying the full model analyzed in Section 3, it will be useful to apply spectral analysis to the simpler problem of evaluating the direct effect, transmitted by the function f, of changes in a marriage series on a parallel birth series. The implications of this link between the series may be explored by calculating the squared gain of f, $|F(\lambda)|^2$. This indicates the extent to which this function (or "filter") amplifies or attenuates the variance in the marriage series at each frequency as it is passed on to the birth series (see [16, 13 or 27]). Thus $|F(\lambda)|^2 g_m(\lambda)$ is the contribution of variance in the marriage series to the variance of the birth series at frequency λ.

Let us begin by considering the effects of very short-run fluctuations in marriages. For example, can seasonal variations in births be explained by seasonal variations in marriages? Several empirical studies have concluded they cannot after comparing the seasonality of first and higher order births (e.g., [15, 34]).

The squared gain function provides an alternate approach. Let F_x^1 be a function giving net *first order* fertility by duration of marriage on a monthly basis, and let $f_x^1 = F_x^1 / \sum F_j^1$. We used data on f^1 for the contemporary U.S. population to calculate this squared gain function and found that at the seasonal frequency of one cycle per year, it equalled .007. Thus, less than one percent of the seasonal variance is passed to first births, and obviously much less would be passed if we considered births of all orders. It is clear, then, that seasonal fluctuations in marriages cannot possibly have much effect on births.

What about longer fluctuations? Figure A shows three squared gain functions for frequencies from 0 to 50 cycles per century. One of these is for first births only, based on contemporary French data (see [28, p. 230]); a second is for all-order births, again based on modern French data (see [28, p. 209]); and the third is for all-order births, based on 18th century data from Crulai, a French parish (calculated from tables in [10]).

A. Squared Gain of the Net Marital Fertility Functions



Source: F_x^1 for France [28, p. 200]; F_x for France [28, p. 230]; F_x for Crulai calculated from tables in [10].

Comparison of the two functions for modern France indicates that first births are about twelve times as sensitive to fluctuations in marriages as are all-order births, over most of the spectrum. Comparison of the functions for modern and 18th century France shows surprisingly little difference, although average births per marriage were about twice as numerous in Crulai as in modern France and life expectancy was less than half as great.[11] Thus it seems safe to generalize from Figure A.

Its most important feature is the rapid decline in squared gain at higher frequencies, indicating that very little variance in marriages is passed on to births except at low frequencies. The functions for all-order births show that for fluctuations of length 20 years, about 1/10 of

[10] To derive cross-spectral results, we need only assume certain *ratios* of $g_\epsilon(\lambda)$, $g_\theta(\lambda)$ and $g_\eta(\lambda)$ at each frequency. These ratios will depend in part on the sizes of the populations studied, since for small populations the fact that marriage rates are much lower than birth rates makes them more volatile.

To derive the spectra of b and m, we need also to specify the *shape* of $g_\epsilon(\lambda)$, $g_\theta(\lambda)$ and $g_\eta(\lambda)$. The assumption of constant spectra (white noise) is an appropriate first approximation, although it is well known that the harvest, real wages and unemployment, each of which affects birth and marriage rates, are autocorrelated and quasi-cyclic.

[11] Recall, however, that both have been transformed to sum to unity.

the variance is passed; at 15 years, this has fallen to 1/20; at ten years, to below 1/30; at five years, to 1/80; and at two years it has fallen below 1/200. From inspection of $|F(\lambda)|^2$, we may conclude that the effect of short-run fluctuations in marriages on births will be almost totally dampened, and longer fluctuations will be substantially attenuated. Thus, it is *in principle* impossible for nuptiality to explain much short-run variation in a birth series. This suggests that the considerable short-run variation that birth series do in fact exhibit must be due to marital fertility.

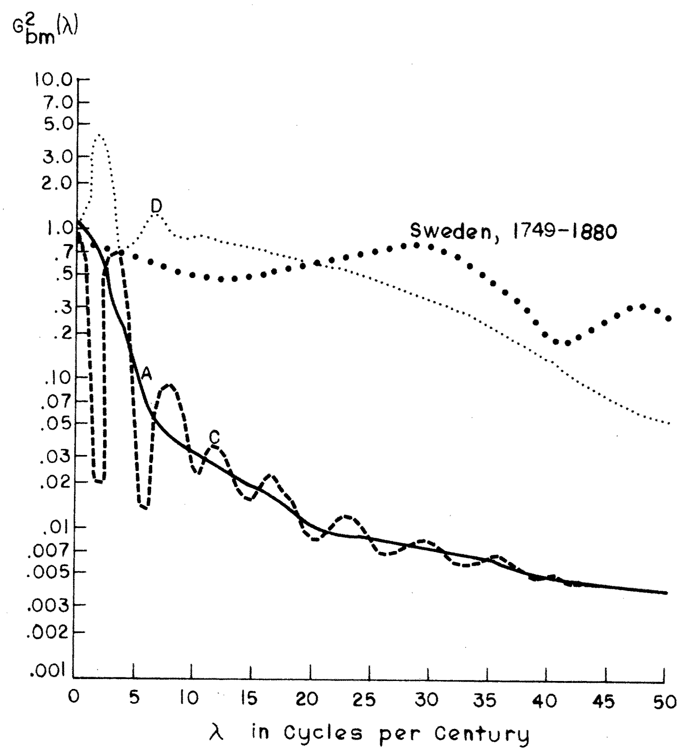
5. THE FULL MODEL AND THE NATURAL FERTILITY HYPOTHESIS

The preceding discussion of the squared gain function casts serious doubt on the strong natural fertility hypothesis even before examination of any actual series. Here we will consider the estimated cross-spectral functions for historical time series in comparison with the implied functions under Hypotheses A, C, and D. (B was omitted because it is not advanced in the literature and would clutter the diagrams. It will be discussed in the section on cycles.) Empirical cross-spectra were estimated for the following populations: five French parishes, including Crulai (1680–1800); Sweden (1749–1880); England and Wales (1780–1870); two English parishes, 1545–1838; Akershus, Norway (1735–1865); U.S. (1920–1970). The results are in each case quite similar, so we will restrict our discussion to the estimates for Sweden.[12]

We begin with the squared gain shown in Figure B. Under Hypothesis A or C, the theoretical squared gain declines very rapidly as frequency increases, reflecting the diminishing influence of marriage fluctuations on births as shown in Figure A. Under Hypothesis D, however, the squared gain remains relatively high; this is not due to the effect of marriages on births but rather to the effect of a common component of variation in nuptiality and marital fertility. The estimated squared gain of births over marriages is seen to conform relatively closely to Hypothesis D and to be completely inconsistent with Hypotheses A and C.

Figure C shows the phase angle, that lag of births behind marriages which produces the maximum correlation at each frequency. Under Hypothesis A, the phase shift is derived solely from the transfer function, $F(\lambda)$, and throughout most of the spectrum this differs little from the phase shift under Hypothesis C. The implied time lag is one year for the shortest fluctuations and rises steadily to six years for fluctuations of 30 years or more. This may be contrasted to the lags implied by Hypothesis D and by the actual Swedish data which are

B. Theoretical and Estimated Squared Gain of Births Over Marriages[a]

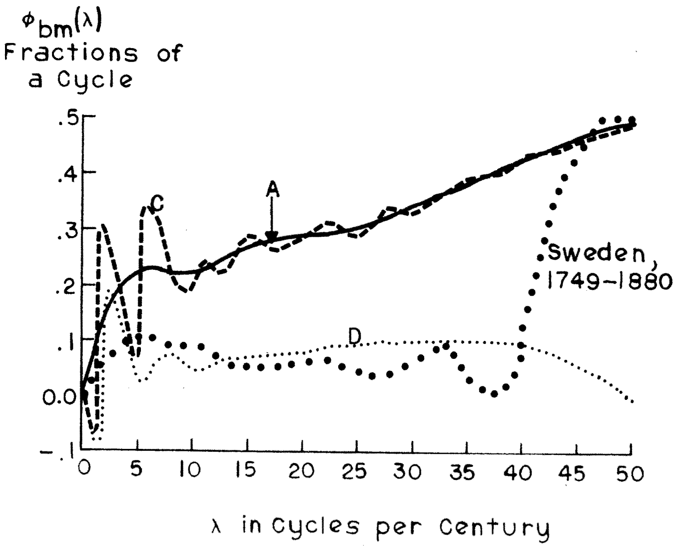


[a] Hypotheses A, C and D have the following interpretation: Hypothesis A: Marital fertility constant. Hypothesis C: Marital fertility and nuptiality vary independently. Hypothesis D: Marital fertility and nuptiality are correlated; lag is 4 months.

Source: For theoretical squared gain, see text. Swedish data taken from [31, pp. 38–44].

much shorter. In fact the short lag estimated from the Swedish data can only be explained by the mechanism incorporated in Hypothesis D: marital fertility and nuptiality are highly correlated and respond to similar

C. Theoretical and Estimated Phase Shift of Births Over Marriage[a]



[a] See Footnote a, Figure B.

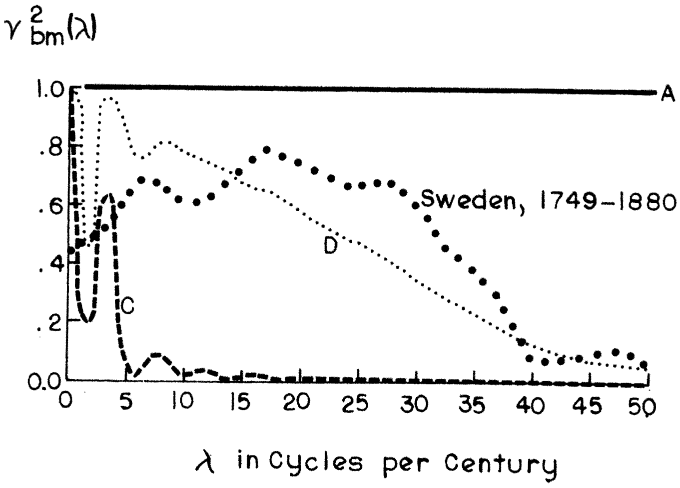
Source: For theoretical phase shift, see text. Swedish data taken from [31, pp. 38–44].

[12] The data for Sweden are taken from [31, pp. 38–44]. Empirical analysis was performed using a program written by Professor Phillip Howrey, employing a Parzen window, which he generously made available to me. There were 132 annual observations, and the estimates were based on 20 lags. For a time domain analysis of these data, see [33]. Sources for the other series were: Crulai [10, pp. 242–4]; Akershus [8, pp. 169–73, 180–4]; England [25]; English parishes—series for Colyton and Hartland generously supplied by Professor E. A. Wrigley.

disturbances; the response of marital fertility lags that of nuptiality by about four months, implying that conceptions must *lead* nuptiality by about five months.[13]

The coherence squared of births and marriages, indicating the proportion of the variance of each that can be explained by their association at each frequency, is shown in Figure D. Under Hypothesis A, all variation in the birth series derives directly or indirectly from variation in marriages, so the coherence squared is 1 at all frequencies. Under Hypothesis C, marital fertility contributes an independent source of variation to the birth series, and as the effect of marriages on births grows weaker at higher frequencies, this independent source of variation comes to dominate; thus, the coherence squared falls rapidly. Also, note the local peak which occurs at a period of roughly 30 years, reflecting

D. Theoretical and Estimated Coherence Squared of Births and Marriages [a]



[a] See Footnote a, Figure B.
Source: For theoretical coherence squared, see text. Swedish data taken from [31, pp. 38–44].

the influence of the generational cycle (which will be analyzed in Section 7) on both the birth and marriage series. Under Hypothesis D, the coherence squared declines approximately linearly over almost the entire spectrum.[14] Since the curve for Hypothesis C shows only the coherence of purely demographic origins, the difference between it and the curve for D represents the additional coherence due to correlation of nuptiality and marital fertility. We see that for frequencies above five per century, nearly all the coherence under D is due to this correlation of disturbances; practically none is due to the demographic effect of marriages on births.

Note that under each of the three hypotheses the coherence is 1 for frequency zero; over very long fluctuations, marriages and births will move together because the size of the base population will be the dominant influence. Once again it is clear that only Hypothesis D provides a reasonable approximation to the estimated relation for Sweden.[15]

We therefore may conclusively reject both the strong and the weak natural fertility hypotheses and are left with the hypothesis that marital fertility and nuptiality were themselves very highly correlated.[16] We cannot say for sure whether this reflects voluntary control of fertility within marriage. However, elsewhere it is shown that in a multivariate cross-spectral analysis, the partial coherence of births with marriages is much stronger than with deaths, casting considerable doubt on the alternative "fecundity impairment" explanation [25]. (For a similar argument, see [2].)

6. SOME RELATED APPLICATIONS

There have been dozens of studies of the effects of business cycles on fertility and nuptiality (e.g., [18, 30]). After removal of low frequency variation using moving averages or fitted polynomials, the typical study finds a high correlation of nuptiality and an economic index, and moderate partial correlations of fertility with nuptiality and the index. In a well-known study, Kirk concludes from such evidence that:

> "Economic conditions control about one-half of the annual variance of fertility from its trend—half of the control operates through nuptiality and the other half is exercised directly on fertility" [18, pp. 253–4].

It should be clear from Sections 4 and 5 that the conclusion is unwarranted and that inclusion of nuptiality in the equation is a misspecification. The detrending operation itself removes nearly all variance in fertility that is not "direct," i.e., not due to marital fertility. This is demonstrated by Figure E which shows $\gamma_{b\epsilon}^2(\lambda)$, the coherence squared of births and marital fertility under Hypothesis C. The period of the business cycle ranges from three to nine years; in this range, γ^2 is over .95, and nuptiality has virtually no influence on births. We conclude that the results of these studies should be reinterpreted and that future studies should avoid this specification.

On the other hand, the high coherence shown in Figure E indicates that raw birth series may be effectively used to study variation in marital fertility, without need for more refined rates which remove the influence of age structure and nuptiality. For example, for fluctuations with period less than 15 or 20 years, the cross-spectrum of a grain price series and a parish register baptism series would measure the influence of prices on marital fertility. This, with annual data, covers 85 to 90 percent of the spectrum. A similar result holds for marriages and nuptiality.

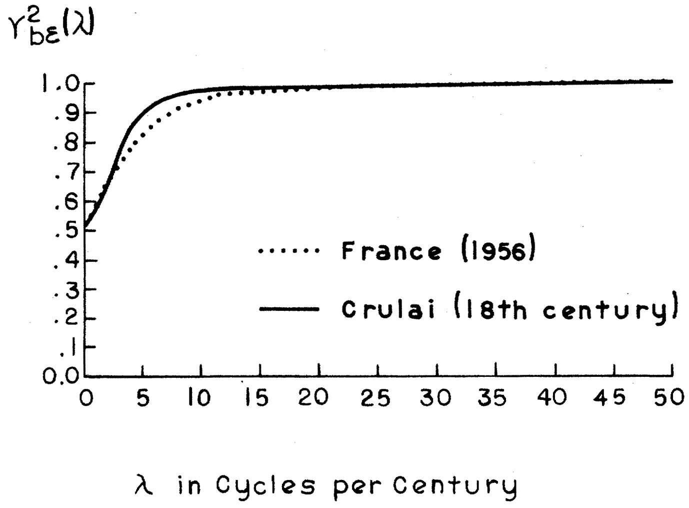
[13] Confidence intervals for the estimates of $\phi(\lambda)$ depend on $\gamma^2(\lambda)$. When the estimated $\gamma^2(\lambda)$ is .5 (see Figure D), the 95 percent confidence interval for $\phi(\lambda)$ is $\pm$.05 (see [16, p. 381]).

[14] This decline is due to the fact that the transfer function $K(\lambda)$ passes increasingly small proportions of their common component of variance, $g_\eta(\lambda)$, as frequency rises. Presumably, with continuous time, this would not occur.

[15] An estimate of $\gamma^2(\lambda)$ greater than .25 is significantly greater than zero at the 95 percent level (see [16, p. 379]).

[16] This result is strongest for large populations. In parish size populations, with one or two thousand people, the pure demographic randomness of births and marriages is more important.

E. Theoretical Coherence Squared of Births and Marital Fertility[a]



[a] Under Hypothesis C: marital fertility and nuptiality vary independently.
NOTE: $1 - \gamma_{b\varepsilon}^2(\lambda) = \gamma_{b\theta}^2(\lambda)$.
Source: For method of calculation, see text. Basic data taken from [28, p. 230] and [10].

7. THE THEORY OF GENERATIONAL CYCLES

Many series of baptisms and marriages seem to move in long waves of 25 to 35 years, a phenomenon often noted by demographic historians [1, 9, 12, 21, 36]. Apparently these are not forced oscillations due to the cyclic movement of a driving series, as with shorter cycles. The forced oscillation hypothesis is rejected, in part, because basic economic series do not reveal appropriate periodicities (see, e.g., [14]) and also because there are other explanations based on the internal structure of the population renewal process.

Common sense suggests that, in the succession of generations, large cohorts will themselves produce large cohorts, after reaching maturity. This argument receives formal expression in mathematical demography and is the classical explanation of generational cycles. Another argument attributes the cycles primarily to the institution of marriage, which imparts inertia (first-order autocorrelation) to the birth process.

The classic analysis is based on the deterministic renewal equation, $B_t = \sum B_{t-a}\phi_a$, or in our notation, $\sum c_i b_{t-i} = 0$. Along a stable growth path, $B_t = e^{rt}B_0$ and $b_t = 0$ for all t. However, when the age structure has been distorted by some shock, the series will oscillate as it converges to stability, and the periods of oscillation can be derived from the complex roots of the equation $\sum c_j y^j = 0$. The dominant period is approximately equal to the mean age at childbearing, which is 25 to 33 years depending on the population [1, 3, 17, 19]. However, this explanation for cycles requires an initial catastrophic shock followed by relatively constant vital rates and even then implies very rapid damping. In fact, observed cycles are not typically preceded by such a single outstanding shock, and the cycles are much too persistent to agree with the theory. The classical explanation is incomplete.

A stochastic version of this analysis avoids these problems. Let $\phi_{a,t}$ be an element of a stationary stochastic vector of age-specific net maternity rates so that the renewal equation becomes: $B_t = \sum \phi_{a,t}B_{t-a}$ (see (2.3)). After the transformations discussed in Section 2, the first-order Taylor approximation may be expressed as:[17]

$$\sum c_i b_{t-i} = \delta_t \; , \tag{7.1}$$

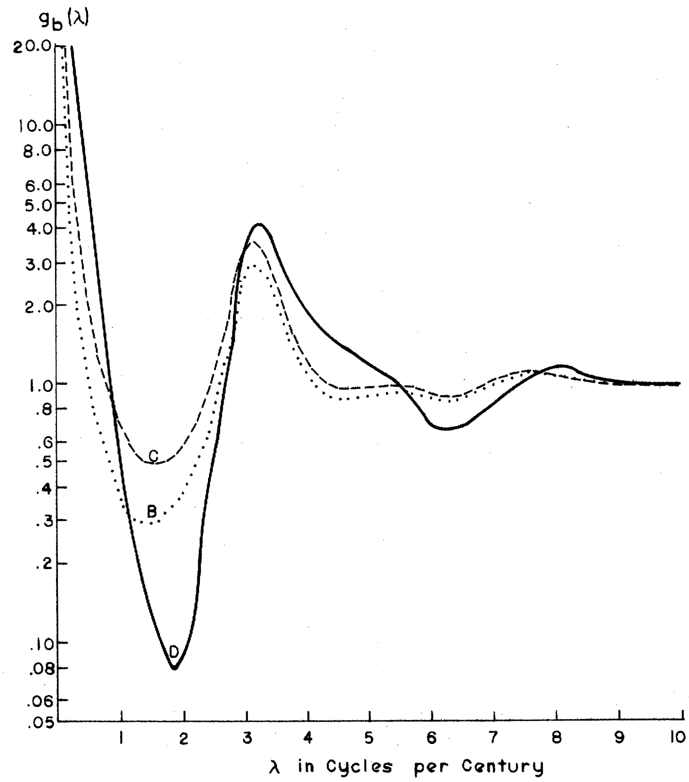
where δ_t is the deviation of the net reproduction rate from unity, or $\sum \phi_{a,t} - 1 = \delta_t$. No distinction is made here between variation in δ arising from nuptiality or marital fertility.

Equation (7.1) implies the following spectral relationship:

$$g_b(\lambda) = \{1/|C(\lambda)|^2\}g_\delta(\lambda) \; . \tag{7.2}$$

The squared gain function, $1/|C(\lambda)|^2$, shows how the age structure of reproduction changes the pattern of variation in vital rates while transmitting it to births. The shape of the squared gain function is shown as Hypothesis B in Figure F. There is a strong peak at a period corresponding to the mean age at childbearing, which shows that uncorrelated random variation (white

F. The Theoretical Spectrum of Births[a]



[a] Hypotheses B, C and D have the following interpretation: Hypothesis B: δ is white noise. Hypothesis C: Marital fertility and nuptiality vary independently. Hypothesis D: Marital fertility and nuptiality are correlated.
Source: For method of calculation, see text. Basic data calculated from tables in [10].

[17] For a more complete discussion, see [24] or [22].

noise) would be transformed into quasi-cyclic generational waves.[18]

This extension of the classic analysis removes the unrealistic assumption of catastrophic shock followed by constant vital rates and shows how the energy which enables the cyclic pattern to persist is drawn from a continuing series of minor disturbances. However we will see that it is still incomplete.

An alternative explanation of the cycles has been offered by Carlsson [2]. He suggested that the net-fertility-by-duration-of-marriage function, f_x, when combined with a moderately autocorrelated disturbance term, imparts sufficient first-order autocorrelation to a birth series to yield an average upcross period of about 30 years. According to this view, "The thirty-year 'period' has nothing to do with the distance between two generations, e.g., between birth and marriage" [2, p. 417].

If this were true, we would expect the autocorrelation functions of actual birth and marriage series to decline monotonically with lag. However, estimated functions show peaks at about 30 years [21]. We conclude that Carlsson's theory is incorrect. The reason is clear: it ignores the feedback in the system. While it correctly stresses that births proceed from marriages, it ignores what is equally correct, i.e., that marriages proceed from births. This amounts to basing the analysis on (2.1), while suppressing (2.2).

Nevertheless, Carlsson is correct that the institution of marriage, by adding persistence to random variation, increases the autocorrelation in the birth series. This accentuates the longer-run fluctuations in births, including the generational cycle, and attenuates shorter-run fluctuations.[19] Thus, the specification of (7.1) does not embody all our prior demographic information about the generating mechanism of the birth series. Taking account of these persisting effects of variations in nuptiality on the net maternity function leads to:

$$\delta_t = \epsilon_t + \sum f_x\theta_{t-x} , \qquad (7.3)$$

which may be confirmed by substituting from (7.3) into (7.1) and comparing the result with (2.5).

The theoretical spectrum of births in terms of the spectra of nuptiality [$g_\theta(\lambda)$] and marital fertility [$g_\epsilon(\lambda)$] now becomes more complicated than (7.2). It has, in fact, already been given in the table under Hypotheses A, B, C and D. Hypothesis A does not interest us at present. Hypothesis B gives the same result as the stochastic version of the classical theory, and the implied spectrum is plotted in Figure F as the dotted line. Hypothesis C combines the classical and Carlsson effects and is plotted as the dashed line in Figure F. It does slightly accentuate the generational cycle, but the effect is not pronounced. Hypothesis D adds the empirically realistic assumption that variations in marital fertility and nuptiality are closely correlated; the implied spectrum is plotted as a solid line. The prominence of the cycle is considerably increased, particularly relative to longer-run fluctuations.[20]

So far we have only discussed birth cycles, but the analysis of marriage cycles is of course completely symmetric and the relevant equations are also shown in the table. The marriage spectra in all cases have somewhat sharper peaks than the birth spectra, showing a stronger cyclic tendency. Under Hypothesis C, the squared coherence at 30 years is .63 with births lagging by six years; under D, the coherence is .96 with a lag of only four years (see Figures C and D).

8. SUMMARY AND CONCLUSIONS

The effect of variation in a series of marriages on the behavior of a parallel series of births was shown to be much weaker than has generally been thought; thus, seasonal fluctuations in births—even first-order births—are virtually independent of the seasonality of marriages. Even with longer fluctuations in marriage series, up to perhaps 15 or 20 years periodicity, there is very little effect on births. It is therefore not surprising that the natural fertility hypothesis, as applied to time series, must be rejected. Short-run fluctuations in preindustrial birth series were due to fluctuations in marital fertility, not to fluctuations in nuptiality. In fact, the coefficients of short-run variation for nuptiality and marital fertility were roughly equal. In addition, fluctuations in nuptiality and marital fertility were highly correlated, suggesting the likelihood of voluntary control of fertility within marriage.

Two related results follow from the analysis: (1) short-run variations in birth and marriage series are excellent proxies for those in marital fertility and nuptiality and (2) the importance of the "indirect" effect of business cycles on fertility, by way of nuptiality, has been greatly exaggerated; in fact, this effect is negligible.

We also developed a theory of generational cycles. A stochastic version of the classical theory, assuming white noise disturbance, accounts for the basic cyclic pattern. The spectral peak at the generational frequency is reinforced by autocorrelation imparted to fertility behavior by the institution of marriage, but this effect is not very important. When we also take account of the high correlation of variations in nuptiality and marital fertility, the cyclic behavior is reinforced still further.

APPENDIX A: THE EFFECT OF AGE AT MARRIAGE AND MORTALITY ON BIRTHS

The model developed in Section 2 emphasizes the effects of changes in the *number* of marriages on births while apparently ignoring the effect of *age* at marriage, a variable frequently stressed in the litera-

[18] The analysis of the stochastic model could be related more closely to the classic deterministic analysis by deriving the autocorrelation function, which involves the same complex roots as the deterministic analysis but with different weights. See [16] or [24].

[19] Evidently fluctuations in mortality have similarly persisting effects, but there is not space here to develop their analysis.

[20] These different specifications also shift the length of the generational cycle slightly. The additional assumption that the disturbance terms (ϵ and θ) have first-order autocorrelation with $\rho = .5$ raises the peak considerably relative to the high frequency spectrum but fails to shift the length of the cycle noticeably.

ture on preindustrial Europe. This calls for an explanation. We will develop two alternative models incorporating changing age at marriage and show that in both cases significant effects on births actually arise only from resulting changes in the number of marriages and not their average age.

Let $n(a)$ be the marriage rate for single females of age a.[21] The first model assumes that nuptiality changes proportionately at all ages by the factor k so that the new nuptiality schedule is: $n^*(a) = kn(a)$. If k is greater than 1, then nuptiality has increased, and the number of marriages (*ceteris paribus*) will also increase. Initially there will be no change in the average age at marriage; women at all ages will marry more frequently. In the long run, the average age at marriage will fall, due to the relatively greater depletion of the stock of spinsters at older ages. But this requires the lapse of considerable time and will not hold for short-run fluctuations in nuptiality. Thus, here the primary short-run effect of an increase in nuptiality is on the number of marriages, not on their average age; this latter will be a second-order effect, of importance only in the long run.

A second model, which seems less plausible, leads to short-run fluctuations in the age at marriage with little variation in the number of marriages. Suppose that the nuptiality schedule shifts towards 0 by d years, but otherwise remains identical, so that the new schedule may be written: $n^*(a) = n(a - d)$. The mean age at marriage is then reduced by nearly exactly d years. This will lead temporarily to more marriages, since the same rates are applied to slightly larger numbers of unmarried (an effect which washes out in the long run) and will lead permanently to a small increase in marriages by avoiding a small amount of mortality. These effects operate through the number of marriages. There is also a pure age-composition effect, since women marrying earlier have more child-bearing years ahead of them. Evaluating, we found that a one-year decline in the average age of a marriage cohort (i.e., $d = 1$) would result in a long-run increase of .15 children per marriage, or four percent. The effect in the short run would, of course, be much less.

The impact of a change in life expectancy at birth, e_0, is also minor. A change of one year in e_0 causes a change of .025 children per marriage, or .6 percent, and again this is a long-run relation. The major impact of mortality change is on numbers marrying, by way of changes in net nuptiality.

In contrast to these rather meager effects of average age at marriage and mortality on births, any change in numbers marrying is transmitted fully (with unitary elasticity) in the long run. It is therefore correct to emphasize changes in numbers marrying in the model.

APPENDIX B: DERIVATION OF THE THEORETICAL CROSS-SPECTRAL FUNCTIONS

Any stationary process, x, which is not strictly periodic has a spectral representation $z_x(\lambda)$ with the property that $E[z_x(\lambda)\overline{z_y(\lambda)}] = g_{yx}(\lambda)$, where $g_{yx}(\lambda)$ is the cross-spectrum of y and x (see [16 or 13]).

Let $z_\theta(\lambda)$, $z_\epsilon(\lambda)$, $z_b(\lambda)$ and $z_m(\lambda)$ be the appropriate representations of the processes θ, ϵ, b and m. Equations (2.9) and (2.10) expressed the relations among these variables in the time domain, in terms of the functions f, w, c. Similarly we may express the relations among the spectral representations of these variables in the frequency domain, in terms of the corresponding transfer functions, denoted $F(\lambda)$, $W(\lambda)$ and $C(\lambda)$. The transfer functions of f_x, for example, are calculated as $F(\lambda) = \sum_x f_x \exp(-ix\lambda)$, where $i = \sqrt{-1}$ (see [16 or 13]). A transfer function times its own conjugate equals its "squared gain"; thus $F(\lambda)\overline{F(\lambda)} = |F(\lambda)|^2$, the squared gain of $F(\lambda)$.

We have, from (2.9) and (2.10):

$$C(\lambda)z_m(\lambda) = F(\lambda)z_\theta(\lambda) + z_\epsilon(\lambda) \tag{B.1}$$

$$C(\lambda)z_m(\lambda) = W(\lambda)z_\epsilon(\lambda) + z_\theta(\lambda) \ . \tag{B.2}$$

[21] This nuptiality rate is related to the net nuptiality distribution used in Part 2 by:

$$w_a = n_a p_a \prod_{i=0}^{a-1} (1 - n_i) \ .$$

The spectra of b and m are derived by multiplying each side of (B.1) and (B.2) by their respective conjugates and taking expected values. Similarly the cross-spectrum of b and m may be found by multiplying each side of (B.2) by the conjugate of the corresponding side of (B.1).

When fluctuations in nuptiality and marital fertility are uncorrelated at all lags, the term $z_\theta(\lambda)\overline{z_\epsilon(\lambda)}$ and its conjugate are zero. More generally, however, they will be nonzero and may be derived from the corresponding spectral representations; thus from (2.11) and (2.12) we have:

$$z_\epsilon(\lambda) = z_{\epsilon'}(\lambda) + K(\lambda)z_\eta(\lambda) \ , \tag{B.3}$$

$$z_\theta(\lambda) = z_{\theta'}(\lambda) + z_\eta(\lambda) \ . \tag{B.4}$$

The cross-spectrum of ϵ and θ will be given by:

$$g_{\epsilon\theta}(\lambda) = \overline{K(\lambda)}g_\eta(\lambda) \ . \tag{B.5}$$

Terms containing $z_\theta(\lambda)\overline{z_\epsilon(\lambda)}$ or its conjugate now have to be included in the derivation of the spectra and cross-spectrum of b and m. These are given by:

$$g_b(\lambda) = (1/|C(\lambda)|^2)\{|F(\lambda)|^2 g_\theta(\lambda) + g_\epsilon(\lambda) + 2g_\eta(\lambda)RE[F(\lambda)K(\lambda)]\} \tag{B.6}$$

$$g_m(\lambda) = (1/|C(\lambda)|^2)\{|W(\lambda)|^2 g_\epsilon(\lambda) + g_\theta(\lambda) + 2g_\eta(\lambda)RE[W(\lambda)K(\lambda)]\} \tag{B.7}$$

$$g_{bm}(\lambda) = (1/|C(\lambda)|^2)\{W(\lambda)g_\epsilon(\lambda) + \overline{F(\lambda)}g_\theta(\lambda) + \overline{K(\lambda)}g_\eta(\lambda) + W(\lambda)\overline{F(\lambda)}K(\lambda)g_\eta(\lambda)\} \ , \tag{B.8}$$

where the function $RE(\ \ )$ is the real part of the complex argument.

From the basic spectral and cross-spectral functions presented in the table, we may derive the gain, denoted $G(\lambda)$, the phase shift, denoted $\phi(\lambda)$, and the coherence squared, denoted $\gamma^2(\lambda)$:

$$G_{bm}(\lambda) = g_{bm}(\lambda)/g_m(\lambda) \tag{B.9}$$

$$\phi_{bm}(\lambda) = \text{Arctan}\ \{IM[g_{bm}(\lambda)]/RE[g_{bm}(\lambda)]\} \tag{B.10}$$

$$\gamma_{bm}^2(\lambda) = g_{bm}(\lambda)^2/[g_b(\lambda)g_m(\lambda)] \ . \tag{B.11}$$

When marital fertility is constant (under Hypothesis A), these take on a simple form. The gain is simply the gain of f, $|F(\lambda)|$; the phase shift is simply that of f, Arctan $\{IM[F(\lambda)]/RE[F(\lambda)]\}$; and the coherence squared equals 1.0 at all frequencies. Similar results hold under Hypothesis B. In general, however, the expressions are much more complicated.

[Received December 1973. Revised November 1974.]

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