Mortality & Covid-19: Stable Theory and Per Capita Deaths

BFDW, Day 2, Lecture 1

Joshua R. Goldstein   Thomas B. Cassidy

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Our Goals

- Learn some classical formal demography (stable theory, life table entropy)
- Use, modify, and extend for Covid

By the end of the day, you should have a good sense of "what and why." Plus, exercises should give you some "how."

Our plan

Hour 1 **Deaths per Capita**
Stable population model, age structure, and per-capita measures

Hour 2 **Effects on Life Expectancy**
Keyfitz's entropy measures elasticity of life expectancy with respect to age-specific mortality

Hour 3 **Beyond Life Expectancy**
Person-years lost as a measure of mortality shocks

Format: lecture (30-35 mins); breakout exercises (25-30 mins); exercise debrief and discussion (10-15 mins).

"Cheat sheet". The whole day in just 3 formulae

Lotka — How crude death rates vary with stable population growth rates

$$\Delta \log CDR(r) \approx (A_0 - e_0)\,\Delta r \tag{1}$$

Keyfitz — How life expectancy changes with proportional change δ in hazards

$$\frac{\Delta e_0^*}{e_0} \approx -\mathcal{H}\delta \tag{2}$$

(New) — How population's remaining person-years change after single-year of proportional change δ in hazards

$$\frac{\Delta \theta_0}{\theta_0} \approx -\frac{\mathcal{H}}{A_0}\delta \tag{3}$$

Stable theory and per capita mortality

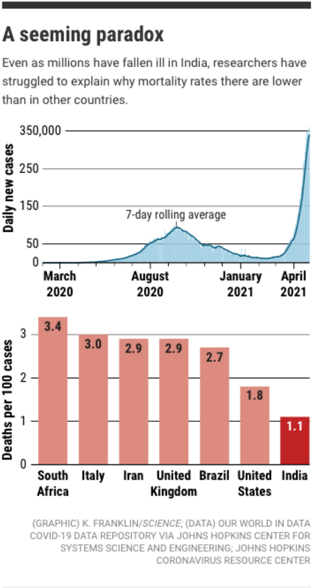
Our question

How does fertility (!?) effect *per capita* Covid death rates

Really, a question about age structure.

- Yesterday, you learned how fertility and pop growth influence age-structure.
- Today, we're going to use stable theory to get a handle on how age-structure influences per capita measures like the Crude Death Rate – or Covid deaths per-person.

India and the United States (from Science)



Explanations:

- Is epidemic not as bad?
- Mortality under-reporting?
- Or, perhaps it's just age-structure?

https://www.sciencemag.org/news/2021/04/will-india-s-devastating-covid-19-surge-provide-da

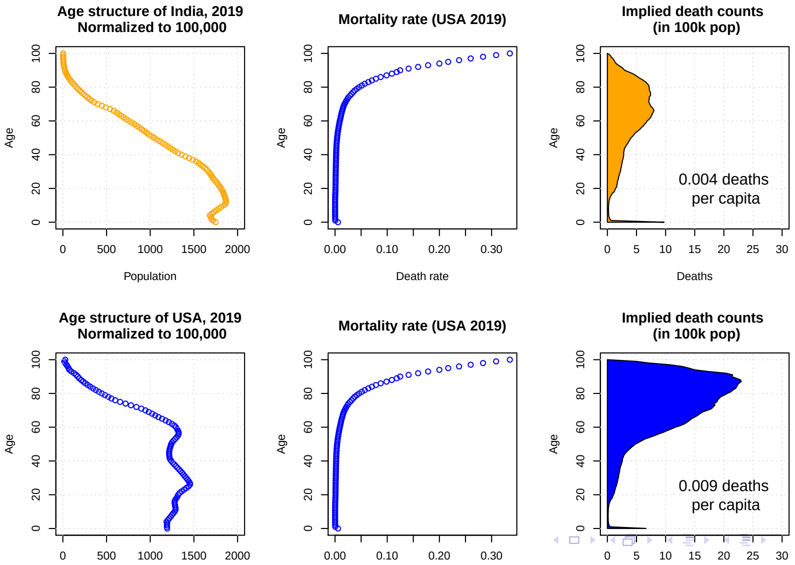
A quote

> *If you have 65% of your population in an age group where mortality rates are extremely low, then obviously, you're going to see an overall case fatality rate that's extremely low. ...*
>
> *[claims of 'India paradox' are] nonsense.*

– Ramanan Laxminarayan, researcher quoted in *Science* article.

An example

Let's see how per-capita death rate varies even when age-specific mortality is the same.



How can we deal with different age structures?

- If we have lots of data, can compare age-specific rates
- Standardization also great. (Direct or Indirect)
- Stable population model can give us a deeper understanding of what drives age-structure dependent measures.
- (And can also be applied to other important quantities that depend on age-structure, like taxes and benefits for a national pension system.)

Age structure of stable pop

For stable pop, you learned yesterday

$$c_r(a) = \frac{e^{-ra}\ell(a)}{\int e^{-ra}\ell(a)\,da}$$

Crude death rate of stable pop

In general

$$CDR = D/N = \frac{\int n(a)h(a)\,da}{\int n(a)\,da} = \int c(a)h(a)\,da$$

In stable pop,

$$\begin{aligned} CDR(r) &= \int c_r(a)h(a)\,da \\ &= \frac{\int e^{-ra}\ell(a)h(a)\,da}{\int e^{-ra}\ell(a)\,da} \end{aligned}$$

Comparative Statics

Now we can do a “comparative statics analysis”: looking at change in $CDR(r)$ with respect to r

How does $CDR(r)$ vary with r?

Easier in logs,

$$\left. \frac{d}{dr} \log CDR(r) \right|_{r=0} = A_0 - e_0$$

$$A_0 = \text{mean age of living in stationary pop}$$

$$e_0 = \text{mean age of death in stationary pop}$$

To derive,

- Write out $\log CDR(r)$, separating numerator and denominator
- Differentiate (careful with minus-signs)
- Stare at and recognize that these are "means"

(Take it away, Tom.)

Interpreting

$$\frac{d}{dr} \log CDR(r) \Big|_{r=0} = A_0 - e_0$$

$$A_0 = \text{mean age of living in stationary pop}$$

$$e_0 = \text{mean age of death in stationary pop}$$

- How big is A_0?
- How big is e_0
- Is the derivative positive or negative? (Does this make sense?)

Approximation for application

Our expression

$$\frac{d}{dr} \log CDR(r)\Big|_{r=0} = A_0 - e_0$$

is exact for infinitesimal changes dr.

For larger changes Δr, relationship is approximate.

$$\Delta \log CDR(r) \approx (A_0 - e_0)\, \Delta r \tag{1}$$

Worked example (1)

- In India, fertility has historically been about 4 children per woman, with pop growth of about 2%.
- In USA, about 2 children per woman, and pop growth closer to 0%.

Worked example (2)

Let's use

$$\Delta \log CDR(r) \approx (A_0 - e_0)\,\Delta r \qquad (1)$$

$$\log CDR_{India} - \log CDR_{USA} \approx (40 - 75)(.02 - 0) = -0.7$$

Exponentiating both sides,

$$CDR_{India}/CDR_{USA} \approx exp(-0.7) \approx 0.5$$

- Not far off from what we got with real age-structures.
- Conclusion: fertility differences can halve per-capita mortality (even when age-specific mortality is same).
- What about Covid?

Application to Covid

$$CDR^{covid}(r) = \frac{\int e^{-ra}\ell(a)h^{covid}(a)\,da}{\int e^{-ra}\ell(a)\,da}$$

$$\frac{d}{dr}\log CDR^{covid}(r) = \text{What?} - \text{what?}$$

Break-Out Exercises

A Calculate crude death rates using simulated stable pops growing at $+2\%$ vs. 0%.

B Calculate using comparative statics result

C Adapt for Covid

Then, we'll regroup and discuss the results.

Discussion (as much as there is time for)

Let's start with your questions . . .

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2. Was comparative statics approximation accurate?
3. Was mean age of Covid death higher than mean age of death? What does this imply about sensitivity of Covid per capita deaths to pop growth?

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Then,

1. For per capita deaths, can pop growth rates make a big difference?
2. Was comparative statics approximation accurate?
3. Was mean age of Covid death higher than mean age of death? What does this imply about sensitivity of Covid per capita deaths to pop growth?
4. Looking back at Science graph, could age-structure be the reason Covid mortality is lower in India than US?

Concluding comments

- Comparative statics calculations with stable pops have lots of applications (how aggregate levels of anything depend on growth rates) and involve mean ages of divorce, paying taxes, widowhood, visiting parks, . . .
- Real pops are not stable, but model is still useful.
- A common strategy here and in next hours will be to use an approximation that is very accurate for small changes, but may not be accurate for larger ones. *Caveat demographer!*